



Understanding the Application of Principal Component Analysis (PCA) and Principal Factor Analysis (PFA) Methods for Construct Validation of Research Instruments.

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Abstract

Measurement instruments for assessment of psychological or social constructs need validity quality to measure truly what they are designed to gauge. In using construct validity to determine the statistical qualities of a research instrument, principal component analysis (PCA) and principal factor analysis (PFA) are essential. The purpose of this paper was to present an overview of the application of principal component analysis (PCA) and principal factor analysis (PFA) methods for construct validation of research instruments. This is because some researchers find it difficult to appropriately use PCA or PFA to determine the construct validity of their research instruments before embarking on data collection despite the fact that it is pertinent for psychological or social construct-based instruments. It was concluded that practitioners should use this article as guide in establishing the validity of their research instruments that are construct-based.

Keywords: Construct-Validation, Measurement, Instruments, Assessment

Introduction

One widely recognized fact about social, behavioral, and educational phenomena is that they are extremely complex. Thus, to understand them better, it is typically necessary to examine many aspects of the various phenomena. Accordingly, many researchers explore different human characteristics and their underlying variables. To understand the multitude of variables, it becomes important to reduce them to more fundamental measures with the property that each of them represents a subset of the initial interrelated variables. For instance, an opinion questionnaire designed by a researcher for data collection may contain numerous items or questions pertaining to several personality characteristics or latent variables. Attempt to reduce such items to a reasonable few, that will substantially explain the latent variables or to make more

meaning out of it brought about the application of the principal component analysis (PCA) and principal factor analysis (PFA).

The Principal Component Analysis (PCA)

Principal component analysis (PCA) is a statistical technique that is specially developed to help in data reduction or to summarize underlying factors of latent variable for clear and better understanding. The PCA does not only allow for data reduction but also carries out data reduction in a manner that permits its results to be used in applications of other multivariate statistical methods (e.g., analysis of variance or regression analysis). In general terms, the major aim of PCA is to reduce the complexity of the interrelationships among a potentially large number of observed variables to a relatively small number of



linear combinations of them, which are referred to as principal components. The interpretation of principal components is generally guided by the degree to which each variable is associated with a particular component to a large extent. The variables that are found to be most closely related to a component in question are used as a guide for its interpretation.

Consider the following empirical example. In a study of characteristics of high-school seniors who continue to college, a researcher collects data on a large number of personality variables, motivation and aspiration measures, general mental ability, scholastic history, family history, health, and physical measures (Tabachnick & Fidell, 2007). The researcher is interested in finding, if possible, more fundamental dimensions that characterize behaviour of students entering college. For instance, some personality characteristics may combine with several motivation and aspiration variables into a derived measure that assesses the degree to which a student can work independently. In addition, several general mental ability measures may combine with some scholastic history to form a derived variable assessing intelligence, etc. As indicated above, these derived measures, called principal components, represent linear combinations of the original observed variables.

This example demonstrates that a PCA of an initial, potentially large set of observed variables aims at “discovering” such linear combinations. The goal of PCA will be achieved if a relatively small number of principal components are “disclosed,” which possess the property that they account for most of the variability in the original data set. If they are also substantively interpretable, it will be an

added benefit that will contribute to better understanding of a studied phenomenon. Consequently, in the behavioural, social and educational sciences, PCA has a relatively long history of application in the development of objective tests for measuring specific abilities, motivation, depression, personality, intelligence, and other related constructs or latent (unobservable, hidden) dimensions. In these applications, one typically starts with a large set of measures aimed at assessing those constructs. Then PCA is employed on the data obtained to reduce the multitude to a few meaningful components that represent “pure” measures of the underlying latent dimensions or variables.

The essence of PCA is therefore the summarization of the pattern of interrelationships so as to obtain linear combinations of subsets of the initial variables. It is desirable that these weighted combinations, which represent the sought components, reduce substantially the complexity of the original data set. Each of them combines a subset of initial variables that are markedly interrelated. In addition, the principal components have the property that they are uncorrelated with one another. Furthermore, each component is obtained in such a way as to account for as much as possible variance in the initial data set. It is thereby hoped that a relatively small number of components account for most of the variance in the original data, and thus accomplish marked data reduction. This is distinct from the central goal of a related statistical technique known as principal factor analysis (PFA) that is majorly concerned with explaining the structure of variable interrelationships.



Principal Factor Analysis (PFA)

The PFA is a statistical technique that shares several similarities with PCA but it is distinct from PCA in several consequential points. PFA was originally developed as a procedure for “disclosing” (discovering) unobserved traits or latent traits, typically referred to as factors, which presumably underlie subjects’ performance on a given set of observed variables and explain their interrelationships. These factors are not directly measurable, but are instead latent or hidden random variables or constructs, with the observed measures being their indicators or manifestations in overt behavior. Statistically, a goal of PFA is to explain the manifest (observed) variable interrelationships. In other words, PFA aims at explaining variable interrelationships (correlations) with as few as possible factors that should be meaningful in a subject-matter area of application. That is the pattern of manifest variable interrelations with a few factors as possible. In essence, the factors are expected to be substantively interpretable and to explain why certain sets (or subsets) of observed variables are highly correlated among themselves.

More specifically, the aims of PFA can be summarized as follows: (a) to determine if a smaller set of factors can explain the interrelationships among several original variables (a similar data reduction concern as in PCA); (b) to find out the number of these factors; (c) to interpret the factors in subject-matter terms; as well as possibly (d) to evaluate the studied persons on the factors, by providing estimates of their factor scores that could be used in subsequent analyses. Furthermore, in PFA the factors are not linear combinations of the observed variables, unlike the principal component analysis (PCA).

Differences between PCA and PFA

- The goal of PCA is to explain observed measure variances with as few as possible principal components that need not be substantively interpretable, while that of PFA is to explain variable interrelationships (correlations) with as few as possible factors that should be meaningful in a subject-matter area of application.
- The principal components (PCA) consider observed factors while the principal factor analysis looks at unobserved factors (PFA).
- In PCA there is no explicit error term, while in PFA there is.
- PCA is essentially assumption-free, while PFA has model-related assumptions.
- In PCA the components are unrelated to one another by construction (since they are defined as orthogonal to each other) whereas in PFA the factors may be related.
- Moreover, in PCA the resulting principal components are linear combinations of the observed variables and thus share the same observed variable status, whereas in PFA the factors are not linear combinations of the manifest measures (even though their estimates are), and are not observed and perhaps not even observable.
- In PCA, because principal components are uncorrelated by definition and are unique, a change in the number of extracted principal components does not change individual scores on the components (that have been extracted before



further components are considered). But in PFA, the factors need not be uncorrelated and are unique only up to a rotation; thus, a change in the number of factors may well alter the estimated factor scores.

Choosing Appropriate Factoring Technique

Different data factoring techniques are available for researchers to choose when running a factor analysis with SPSS. For instance, SPSS version 21 displays these factoring techniques to include; unweighted least squares, generalized least squares, maximum likelihood, principal axis factoring (PAF), alpha factoring, image factoring and principal component analysis (PCA). Similarly, Tinsley and Tinsley (1987) listed the techniques as; the principal component analysis, the principal factor analysis (principal axis factoring) image covariance analysis (image factoring) and exploratory factor analysis. Of these, the principal component analysis and principal factors analysis are the commonly used methods and are usually assumed to produce similar results.

The principal component analysis (PCA) is a method of exploratory factor analysis (EFA) to be employed “when researchers are interested in reducing the number of items in an instrument to the fewest number that best measure the constructs understudy” (Courville Thompson, 2001). The PCA maximizes all variance in the items in such a way that items with no contribution or little explained variances are evaluated for deletion. Whereas, the principal factor (PFA), is a technique that researchers may use when they are interested in determining the underlying factors related to a set of items (Field, 2000). The PFA restricts the common variance among variables. In other words,

the PFA cannot redistribute the variance that is peculiar to a given variable. However, the choice of these techniques depends largely on what the researcher is interested in or the essence of the factor analysis.

Supposing a researcher wants to measure “Lecturers’ Financial-induced Stress in COVID 19 Era” and has developed fifty (50) items or statements to form the instrument, he/she needs to establish the construct-validity of the instrument. The name of this instrument could be called “Lecturers’ Financial-induced Stress Scale (LEFISS-C19)”. The researcher has to trial-test the instrument on a sample of teachers drawn from their population. Assuming 100 copies of the instrument were produced and administered to a sample of 100 respondents. The responses will then be subjected to factor analysis on SPSS. On this note, the first thing the researcher needs to know is that there are different data factoring techniques on SPSS for ensuring that the items or statements are related to the construct (Financial-induced Stress) understudy or the latent variables that account for “Lecturers’ Financial-induced Stress. If the researcher intend to reduce the items or statements to the fewest number that will best measure the construct, then the principal component analysis (PCA) is the appropriate factoring technique. This is because the PCA maximizes all variance in the items in such a way that items with no contribution or little explained variances are discarded, which brings about reduction in the number of items in the instrument to the fewest number that will best measure the construct. Whereas, if the researcher’s interest is to determine the underlying dimensions or factors related to the items or statement on the construct (Financial-



induced Stress), then the principal factor Analysis becomes the best factoring technique to be adopted.

Choice of Minimum Number of Factors

It is noteworthy that not all factors are retained in factor analysis. After extraction, the researcher needs to choose the minimum number of factors to retain for rotation in the analysis. This is because the number of factors retained during factoring can influence the result or output of the analysis. For this reason, Cattell (1966) advocated the plotting of a graph (scree plot) of each eigenvalue (Y-axis) against the factor (component) with which it is associated (X-axis). The number of factors to be retained using a scree plot are those above the “inflexion point” or elbow point. According to Stevens (2002), with a sample of more than 200 participants, the scree plot provides a fairly reliable criterion for factor selection. That is to say, consideration must be given to the sample size when drawing conclusions on results of factor analysis that are based on the scree plot extraction of factors.

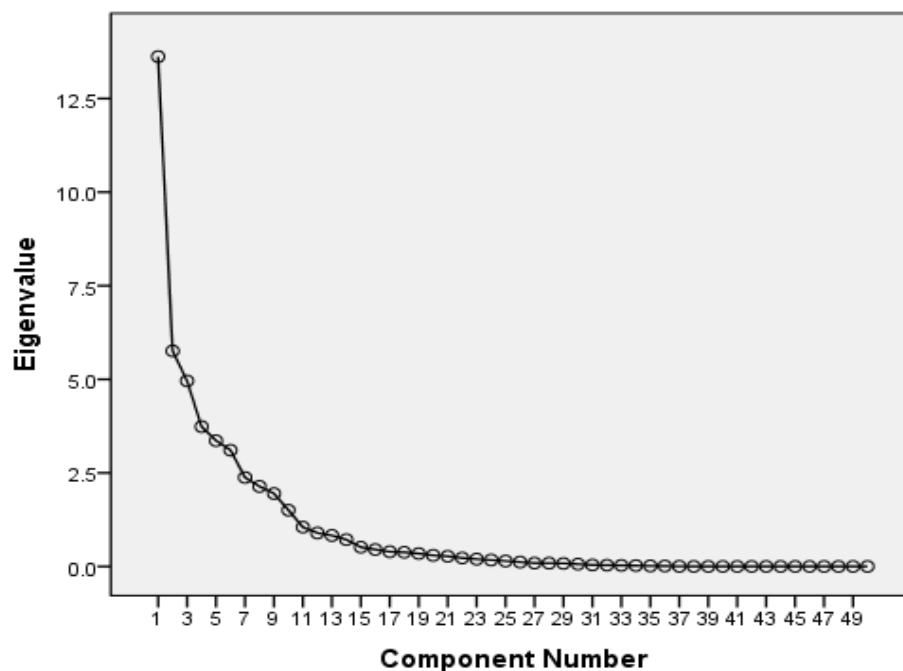
Notwithstanding, Kaiser (1960) recommended retaining all factors with eigenvalues greater than 1. This criterion is based on the idea that the eigenvalues represent the amount of variation explained by a factor, and more so an eigenvalue of 1 represents a substantial amount of variation. Meanwhile, Jolliffe (1986) asserted that the Kaiser’s criterion is too strict and suggested retaining factors with eigenvalues more than 0.7 and also that, although the scree plot is very useful in the selection of minimum number of factors, selection should not be based on this criterion alone. Accordingly, Field, (2009)

suggested that the scree plot in conjunction with the eigenvalue’s criterion should be used to determine the number of factors to retain. However, oftentimes, the number of factors researchers may choose based on these criteria may produce factors with very few items that the researcher may not likely retain. On this note, the researchers have to rerun the analysis until the minimum number of factors that will perfectly measure the underlying construct or yield optimum result is realized for proper construct-validation of their instruments.

Based on the above example, supposing the researcher’s interest is to reduce the fifty (50) items or statements to the fewest number that will best measure “Lecturers’ Financial-induced Stress”, and has chosen the principal component analysis (PCA) which is always the default factoring technique, the researcher has to critically and reasonably choose the minimum number of factors with which the analysis will be run since the output of the analysis will be greatly influenced by the number of factors adopted during factoring. The minimum number of factors to be retained can either be determined using the scree plot or eigenvalue as earlier mentioned. With the data set collected from a sample of 100 respondents as earlier indicated, using the Scree plot, the researcher will have to extract and retain all the 6 factors above the “elbow” or “inflexion point”. Or using the eigenvalues criterion will retain the 11 factors with eigenvalues greater than 1. Below is the scree plot showing the eigenvalue and component number, and the total variance explained by each of the 50 components (items) obtained from the factor analysis using the principal component factoring technique.



Scree Plot



Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	13.621	27.242	27.242	13.621	27.242	27.242	6.916	13.832	13.832
2	5.758	11.516	38.758	5.758	11.516	38.758	6.908	13.815	27.647
3	4.958	9.916	48.674	4.958	9.916	48.674	6.277	12.555	40.202
4	3.736	7.471	56.145	3.736	7.471	56.145	5.815	11.630	51.832
5	3.359	6.718	62.863	3.359	6.718	62.863	5.516	11.032	62.863
6	3.105	6.211	69.074						
7	2.378	4.757	73.831						
8	2.139	4.277	78.109						
9	1.948	3.896	82.005						
10	1.503	3.007	85.012						
11	1.050	2.099	87.111						
12	.896	1.792	88.903						
13	.828	1.656	90.558						
14	.717	1.433	91.992						
15	.514	1.028	93.019						
16	.452	.904	93.924						
17	.395	.789	94.713						
18	.379	.758	95.471						
19	.345	.690	96.161						
20	.297	.594	96.755						
21	.272	.545	97.300						
22	.228	.456	97.756						
23	.196	.392	98.148						
24	.174	.347	98.495						
25	.144	.289	98.784						
26	.118	.235	99.019						
27	.093	.185	99.204						



28	.088	.176	99.381					
29	.082	.164	99.545					
30	.064	.128	99.673					
31	.041	.081	99.754					
32	.037	.073	99.827					
33	.028	.056	99.883					
34	.025	.050	99.934					
35	.015	.030	99.963					
36	.013	.026	99.990					
37	.005	.010	100.000					
38	.000	.000	100.000					
39	1.001E-013	1.003E-013	100.000					
40	1.001E-013	1.002E-013	100.000					
41	1.001E-013	1.001E-013	100.000					
42	1.001E-013	1.001E-013	100.000					
43	1.000E-013	1.001E-013	100.000					
44	1.000E-013	1.001E-013	100.000					
45	1.000E-013	1.000E-013	100.000					
46	-1.000E-013	-1.000E-013	100.000					
47	-1.000E-013	-1.001E-013	100.000					
48	-1.001E-013	-1.002E-013	100.000					
49	-1.001E-013	-1.002E-013	100.000					
50	-1.002E-013	-1.003E-013	100.000					

Extraction Method: Principal Component Analysis.

With reference to the above outputs, 6 factors can be extracted and retain for rotation based on the scree plot, and 11 factors based on the eigenvalue. The 6 factors based on the scree plot explain or accounts for 69.1 percent of the total variance in the data set, whereas, the 11 factors explain or accounts for 87.1 percent of the variance. However, because usually the scree plot and eigenvalue criteria may produce some factors that may not be retainable due to a few number of items associated with such factors, the researcher must continue to rerun the analysis until the minimum number of factors with the maximum number of items emerges. It is in this light that the above analysis was run with 11, 10, 9, 8, 7, 6 and 5 factors. Based on the results, it is with 5 factors that the minimum number of factors with the

maximum number of items was obtained. Hence, 5 factors were retained for rotation during the actual factoring.

Choosing an Appropriate Rotation Technique

In factor analysis, the essence of rotation is to simplify and clarify the structure of the data set. Usually, it is common to find variables in a data set with high loadings on the most important factor and small loadings on other factors. This makes interpretation difficult, which calls for rotation of factors in order to reveal the underlying dimensions of the factors. There are two types of rotation a researcher may adopt when running a factor analysis. There are the orthogonal and oblique rotations. The orthogonal (unrelated) rotations assume that the factors in the analysis are uncorrelated, thus, rotates the factors while



keeping them independent or uncorrelated thereby producing unrelated factors. In contrast, oblique rotation techniques assume that the factors are related, as such; factors are allowed to correlate with each other. According to Field (2009), the choice of a rotation technique depends majorly on whether or not there is a good theoretical reason to suppose that the underlying factors are related or independent. Therefore, if a researcher expects the factors to be unrelated, the orthogonal rotation should be employed, whereas if, there are theoretical grounds to suppose that the factors might correlate with each other, then the oblique rotation should be adopted. In this regard, Pedhazur and Schmelkin (1991) had suggested that if the oblique rotation demonstrates a negligible correlation between the extracted factors, then it is reasonable to use the orthogonal rotation technique, otherwise oblique rotation should be upheld.

Nonetheless, in running a factor analysis with SPSS, three methods of orthogonal rotation- (varimax, quartimax and equamax) and two methods of oblique rotation-(direct oblimin and promax) are available for utilization. These methods however differ in how they rotate the factors which is likely to influence the outputs. For instance, in orthogonal rotations, varimax seeks to minimize the dispersion of loadings within factors, while quartimax attempts to maximize the spread of factor loadings for a variable across all factors. Whereas, equamax is a hybrid of

the other two approaches and is reported to behave fairly erratically (Tabachnick & Fidell, 2007). But, in oblique rotation, the direct oblimin ascertains the degree to which factors are allowed to correlate by the value of a constant called "delta", whereas promax is a faster procedure designed for very large data set. However, Field (2009) maintained that there are strong grounds to believe that orthogonal rotations make complete nonsense of naturalistic data; especially data involving human psychological constructs since there are related theoretically. In other words, psychological and social construct are reasonably thought to correlate with each other in one way or the other. In this respect, researchers can only ensure the construct validity of their instruments when an appropriate rotation technique is employed.

Thus, with the above example, the researcher has three orthogonal - (varimax, quartimax and equamax) and two oblique - (direct oblimin and promax) rotation techniques to choose from. And since these techniques differ in how they rotate the factors, they are likely to produce different results. That is why the researcher must ensure that the appropriate rotation technique is adopted to maximize the result of the output. Thus, rotating the data set in our example with Varimax (orthogonal) rotation and Direct oblimin (Oblique) rotation produced results with varied number of surviving or factorially pure items and factor loadings as shown below;



Varimax (Orthogonal) Rotation

Factor 1		Factor 2		Factor 3		Factor 4		Factor 5	
Item	Loading	Item	Loading	Item	Loading	Item	Loading	Item	Loading
item1	.698	item9	.718	item2	.850	item7	.884	item5	.863
item4	.539	item11	.837	item13	.539	item8	.771	item18	.863
item10	.643	item22	.747	item15	.850	item25	.750	item23	.792
item20	.669	item24	.631	item37	.850	item42	.884	item29	.663
item26	.605	item38	.588	item50	.831	item43	.771	item40	.863
item32	.553	item46	.728						
item36	.634								
item49	.662								

Direct oblimin (Oblique) Rotation

Factor 1		Factor 2		Factor 3		Factor 4		Factor 5	
Item	Loading	Item	Loading	Item	Loading	Item	Loading	Item	Loading
item2	.712	item5	.894	item7	.887	item1	.712	item11	.554
item3	.858	item18	.887	item8	.805	item4	.568	item22	.701
item9	.686	item23	.763	item25	.780	item10	.643	item24	.550
item12	.840	item29	.582	item42	.687	item17	.568	item28	.662
item15	.712	item40	.894	item43	.765	item20	.677	item46	.573
item34	.825					item26	.608	item48	.679
item37	.712					item32	.549		
item38	.766					item36	.652		
item44	.533								
item47	.840								
item50	.677								

Based on Steven's (2002) recommendation that factor loadings should be greater than 0.512 for a sample size of 100, using varimax (orthogonal) rotation, the output on the "Rotated Component Matrix showed that 8 items loaded to factor 1, 6 items to factor 2, and 5 items each to factors 3, 4 and 5, making a total of 29 surviving or factorially pure items, out of the initial 50 items generated. Whereas, using direct oblimin (oblique) rotation, the output on the Structure Matrix which shows the rotated result indicated that 11 items loaded to factor 1, 5 items to factor 2, 5 items to factor 3, 8 items to factor 4 and 6 items to factor 5, making a total of 35 surviving or factorially pure items. These 5 factors explained 62.86 percent of the total variance present in the data set.

Conclusion

Generally, multivariate statistics can be conceptualized as a set of methods that deal with the simultaneous analysis of multiple outcome or dependent variables (DVs). Some multivariate statistical techniques are often concerned with a substantial number of correlated variables. The principal component analysis (PCA) is a dimension-reduction tool that can be used to reduce a large set of variables to a small set that still contains most of the information in the large set. It is a mathematical procedure that transforms a number of (possibly) correlated variables into a (smaller) number of uncorrelated variables called principal components. That is to say, it looks for a set of factors that can account for the total variability in the original variables. This is unlike the principal



factor analysis which tries to find the lowest number of factors that can account for the variability in the original variables that is associated with these factors

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