

Learning Difficulties in Mathematical Problem-Solving at Pre-Tertiary Levels

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Abstract

Problem-solving is key to learning mathematics at all educational levels as it enhances deeper mathematical understanding and transfer of mathematical knowledge to both familiar and unfamiliar situations. It is quite unfortunate that mathematical problem-solving still remains a challenge for students across the different educational levels despite its importance. Many students still encounter difficulties at each step of the general problem-solving process prescribed by Polya. These difficulties include: lack of proper comprehension of the problem statement; lack of strategic competence; inability to implement an appropriate plan; and evaluation and verification difficulty. Three contributory factors to these difficulties in problem-solving were identified which include: knowledge factors; control factors; beliefs and affective factors. It is therefore recommended that teachers should take cognizance of these difficulties and factors contributing to them in order to devise means of helping students overcome such challenges. This will develop their problem-solving skills and make them good problem solvers.

Keywords: Learning difficulties, Problem-solving, Mathematics, Educational levels

Introduction

In life, we encounter and resolve problems each day as we struggle for survival. Problem-solving has two root words namely: “problem” and “solution”. The first part is the existence of a problem, that is, a challenge which must be tackled but has no visible solution and the other part entails structuring the problem and finding various ways of resolving it (Skills YouNeed, 2022). The major goal of any mathematics teaching is problem-solving. Problem-solving learning in mathematics enables learners “acquire the ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom” (NCTM, 2000 as cited in Kennedy, Tipps & Johnson, 2004, p. 134).

Problem-solving is not just being able to solve many computation exercises but to ensure that the child sees the relationship between an event and its mathematical model/representation and also establish a relationship between daily occurrence and the mathematical representation of such. It becomes important to provide children with carefully selected and significant problems that will enable them see this relationship. Thus, Palmer and Bommel (2018) maintained that children’s mathematics education should be focused on problem-solving rather than computation exercises to facilitate creativity in solving problems. To do this, there is the need to develop problems that are derived

from real world situations and events that are sensible to the child.

Nigeria’s education system comprised of three different levels: basic education (nine years), post-basic/senior secondary education (three years), and tertiary education (four to six years, based on the course of study). The pre-tertiary levels comprised the basic education covering six years of primary education and three years of junior secondary education, while the post-basic education covers three years of senior secondary education (World Education Services (WES), 2022).

It is quite unfortunate that despite the various ways and processes that could be utilized in solving problems, students in basic, senior secondary and even tertiary still find it difficult to tackle some mathematical problems when confronted, reason for this paper. Thus, the following are considered in this paper:

- a) Context of problem in mathematics
- b) Context of mathematical problem-solving
- c) Mathematical problem-solving process
- d) Learning difficulties in mathematical problem-solving at pre-tertiary levels
- e) Factors contributing to students’ difficulties in mathematical problem-solving

Context of Problem in Mathematics

Problem is simply a difficulty that has to be resolved or dealt with, that is, a puzzle, question, etc. set for

solution (Collins dictionaries online, 2022). Novriani and Surya (2017) explained a problem to be a situation in which the solver initially does not know any algorithm or procedure that will guarantee solution but the individual desires to solve it. By this explanation, problem is any situation confronting a person without any solution method in view. In other words, problem exists when the solver has a goal but has no initial knowledge of achieving the goal (Mayer, 2022). It should be noted that problem is personal and relative, that is, a student's problem might not be a problem for another. It is dependent on the solver's knowledge, level, ability, experience and time (Mayer, 2022). For example, " $6 + 7 = \underline{\quad}$ " might be a problem for a six-year old Basic one pupil and not a problem for a student in Basic seven.

Owing to differences in knowledge, experience, ability or interest, a problem for one person may not be a problem for another. Also a problem for someone at a particular time may not be so at another time. In some contexts, as students develop their mathematical ability, what were problems initially may after some practice become mere exercises. (Kaur, 1997, p. 95).

Basically, problems can be classified into two: ill-defined problems and well-defined problems. Ill-defined problems are problems not properly defined, without clear starting point, goals, solution paths or expected solution. The solver is left to determine the starting using the features of the problem, the operators to utilize, the specific goal and the rule to follow in getting the problem solved. Well-defined problems are problems that are clearly and properly defined, with known starting point and end point. They have definite specific goals, clearly defined solution path and clearly expected solution (Arifin, Zulkardi, Putri, Hartono & Susanti, 2017; Willingham & Riener, 2019).

Problem in mathematics can be referred to as a task in which a student is interested in resolving without any "readily accessible means" of resolving it (Schoenfeld, 1989 as cited in Kaur, 1997). They are solved by linking prior knowledge to producing a solution in a novel way (Laine, Naveri, Ahtee & Pehkonen, 2014). Thus, a mathematical problem is a task that activates students' thinking and requires the application of productive way of thinking to solve (Oldridge, 2017). According to Laine, Naveri, Ahtee & Pehkonen (2014), this kind of mathematical problems may be either standard/ routine problems or non-standard/

non-routine problems. Standard/ Routine problems are exercises whose solution procedures can be easily recognized based on prior knowledge about them which are dependent on the time the task is given and the level of the person solving the problem. To a school beginner, a task can be a problem if the task is a new task that has not been done before but to those that are already familiar with the task, it becomes a routine task to them. They could be resolved by using a formula, an equation or any method previously known in order to connect between mathematical knowledge and the real world (Saygili, 2017). Non-standard/ Non-routine problems are tasks requiring new kinds of thinking from students because they are presented in an uncommon/unusual way demanding logical and abstract reasoning (Laine, Naveri, Ahtee & Pehkonen, 2014), their solutions cannot be guessed in advance and cannot be resolved with a method or formula previously learnt or known (Saygili, 2017) but require upper-level thinking skills and reasoning to resolve. They are essential in developing problem-solving skills, critical thinking and creative skills. Examples are puzzles. Puzzle as described by Badger, Sangwin, Hawkes, Burn, Mason and Pope (2012), is "a question that is free-standing, light weight and needing ingenuity to solve it". It is a question that is not based on any particular mathematical concept or content, requiring little or no prior knowledge, having many solution paths and strategies, and is in itself interesting. Being lightweight suggests that it may not have a wide impact on mathematical knowledge but could be utilized in later context like unfamiliar situation. It arouses students' interests and curiosity and requires creativity on the part of the learners to arrive at a satisfying solution and also challenges the intellect of students and makes them responsible for their learning.

Mathematical tasks are seen as having starting point, end point and solution path and they can be further categorized into either closed or open tasks. Close tasks are those whose starting and end points are uniquely determined via one solution route as presented in most mathematics textbooks. Contrarily, open tasks are not specifically defined, having many solution methods/ strategies with multiple correct solutions to the problem. They require many possible alternative routes for the starting and end points to arrive at the solution and often require creativity on the path of the solver, that is, the ability to produce new knowledge and flexible problem solving abilities (Hahkionemi, Lapaaho & Fransisco, 2011).

Context of Mathematical Problem-Solving

Problem-solving is defined as “the means by which an individual utilizes previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation” (Krulik & Rudnik, 1980 as cited in Carson, 2007, p. 7). Woods, et al. (1997, p. 75) defined problem-solving as “the process used to obtain a best answer to an unknown, or a decision subject to some constraint”. Toffler (2019) defined problem-solving as the ability to identify the key questions in a problem, develop possible plans for solving, follow through on those plans, and evaluate both the success of the plan and the solution. The following five rubrics were identified as guide to problem solving: identify what is known; define the problem; generate possible solution strategies; apply problem solving steps; and evaluate solution. Simply, problem solving is a process of resolving any category of problem.

Problem-solving in mathematics is regarded as the type of mathematical knowledge which is not limited to the ability to apply facts, rules and algorithms (Jacinto & Carreira, 2012) but involves mathematical thinking and competence utilized in solving a problem which makes a good problem solver. Mathematical problem-solving is viewed as what a solver does in tackling a problem entailing getting an understanding about the problem, thinking critically about how to go about the problem, devising the plan to solve the problem by searching for appropriate strategy to utilise, implementing the appropriate strategy, and reflecting on the solution obtained to ensure it is an optimal solution. It is an activity carried out by a solver to arrive at the solution to a problematic task, hence bridging the gap between the outset of a problem and the solution. It entails various cognitive activities which requires both routine and non-routine pre-learnt knowledge and skills (Jacinto & Carreira, 2012; Lesh & Zawojewski, 2007; Lester & Kehlan, 2003 as cited in Manouchehri, Zhang & Liu, 2012, Miller & Baner, 2012).

Problem-solving is key to learning mathematics. Effective and meaningful mathematics instruction requires a common understanding of the constituents of mathematical problem-solving process and its applicability in tackling problems. It is viewed by many researchers as an essential medium of enhancing deeper mathematical understanding, hence it should be taught to all students at all levels (Chamberlain, 2008; Carson, 2007; Laine, Naveri, Ahtee, & Pehkonen, 2014). Some reasons for teaching problem-solving in

mathematics as given by Laine, Naveri, Pehkonen, Ahtee and Hannula (2012) include the following four categories: problem-solving develops general cognitive skills; it supports the development of creativity; it is a part of mathematical application process (heuristics); and it motivates students to learn mathematics.

Thus, problem-solving teaching has various forms and can be viewed from three major perspectives: teaching through problem-solving which implies teaching problem-solving as a method of learning mathematical content; teaching about problem-solving which implies teaching problem solving as a topic in order to get acquainted with the problem-solving processes; and teaching for problem-solving which implies teaching problem-solving in order to be able to apply it in practical situation (Nunokawa, 2004 as cited in Piggot, 2011).

Mathematical Problem-Solving Process

Problem-solving process in mathematics is such that enables students to utilize their prior knowledge in pattern exploration, conjecture making, hypotheses testing, reflecting and applying pre-learnt concepts to explain and justify reasons to the solution of a problem in mathematics (Francisco & Maher, 2005). This paper used Polya’s idea of general problem-solving process. In an attempt to solve mathematical problems, Polya (as cited in Rott, 2012) prescribed in his book titled “How to solve it” published in 1945, a four-step linear model for problem-solving process, to guide the solution of a mathematical problem and to aid the achievement of an optimal solution.

The four-step problem-solving process model include: understanding the problem, devising a plan, carrying out the plan and looking back. The first step is simply the stage of identifying the key words and interpreting the given problem. After interpreting the problem using the key words from the problem, the next step is to identify and determine the appropriate and useful strategy of finding a solution to the problem (Atteh, Andam & Obeng-Denteh, 2017) such as looking for patterns, making a table, making an organized list, drawing diagrams, writing mathematical equations, reasoning, simplifying problems, etc. The third step entails the application of the strategy identified in step 2 to solve the problem, based on the available information (Rudder, 2006). The concluding step entails evaluating the reasonableness of the answer and making appropriate generalization. If the strategy used is found not valid, alternative strategies will be tried

and utilized to arrive at the optimal solution to the problem (Atteh, Andam & Obeng-Denteh, 2017). This consolidates knowledge and develops the ability to solve problems and justify the solution got (Rudder, 2006).

Learning Difficulties in Mathematical Problem-Solving at Pre-Tertiary Levels

A common view among many is that, doing mathematics is solving problems and mathematics is about how to solve problems (Novriani & Surya, 2017; Rott, 2012). This accorded greater importance and recognition to problem-solving in mathematics curriculum in many countries including Nigeria. By implication, problem-solving is a major reason for studying mathematics because it builds in students the ability to apply mathematics knowledge to various situations in life, either familiar or novel situations (Kaur, 1997). Despite the importance accorded to problem-solving in the curriculum, mathematical problem-solving tasks still remain a challenge for many students across the different educational levels (Noprianilubis, Panjaitan, Surya, & Syahputra, 2017; Novriani and Surya, 2017). Novriani and Surya (2017) found in their study that students have difficulty solving mathematical problems in reading question texts, interpreting the problem, understanding the problem and thereby guessing the answer. Most times, students commit errors by wrong interpretation of problem statements using wrong numerical values leading to incorrect solutions or results (Siniguan, 2017). Thus, learning difficulties can be encountered at each stage of Polya's general process of problem-solving.

Siniguan (2017) outlined these common difficulties to include the following: lack of proper comprehension of the problem statement; lack of strategic competence; inability to implement an appropriate plan; and evaluation and verification difficulty.

- a) Lack of proper comprehension of the problem statement: This is the inability to properly understand the problem posed, identify the key words in a problem, and distinguish the relevant and irrelevant information, thus impeding the accurate interpretation of the problem.
- b) Lack of strategic competence: The inability to competently devise a plan of solving a problem, that is, the inability to identify and select the appropriate strategy from many alternative strategies.

- c) Inability to implement an appropriate plan: The inability to translate a problem into a mathematical form and solve, that is, the inability to implement the appropriate plan or strategy to solve a problem.
- d) Evaluation and verification difficulty: A difficulty encountered when students lack the ability to establish the correctness of the problem solved.

Siniguan (2017) therefore stressed the need for teachers to take cognizance of these difficulties and factors contributing to difficulties encountered by students in order to be able to devise the means of helping students overcome them so as to develop their problem-solving skills adequately and make them become good problem solvers.

Factors Contributing to Students' Difficulties in Mathematical Problem-Solving

Kaur (1997) identified and discussed three cognitive factors and affective factors contributing to students' difficulties in problem-solving, they include: knowledge factors; control factors; and beliefs and affective factors.

1. **Knowledge Factors:** Students' lack of some essential and relevant mathematical knowledge and the coordination of the appropriate skills that will improve their problem-solving ability contribute to the difficulties encountered in solving mathematical problem-solving. These relevant types of knowledge essential for understanding mathematical problem-solving and development of problem-solving skills include: linguistic knowledge, schematic knowledge, conceptual knowledge, strategic knowledge and algorithmic knowledge. Linguistic knowledge is the ability to read, understand problem language and decode the problem statement for easy representation that will enable proper understanding of the problem. Lack of linguistic knowledge poses a difficulty to learning mathematical problem-solving as students misinterpret and misrepresent the given problem. Students must have the ability to read, understand the language of a problem and decode the problem statement for proper understanding of the problem task (Siniguan, 2017). Schematic knowledge has to do with the prompt familiarity with the structural features of

mathematical problems and the problem types. The inability to recall information about the structure of the previously solved problems results in focusing on the surface features which impedes the linking of previous knowledge to new problem situation, thereby posing an obstacle to learning mathematical problem-solving successfully. Conceptual Knowledge entails the content knowledge and the identification of the basic mathematical facts, concepts and principles needed to perform mathematical operations and solve problems. A lack of conceptual knowledge results in challenge encountered in mathematical problem solving. Strategic Knowledge directs the approach students take to resolve a problem. It entails the ability to identify techniques and various useful strategies needed to develop plans to solve mathematical problems. Lack of the knowledge of strategies also contributes to difficulties encountered in learning mathematics problem-solving. Algorithmic knowledge which is also referred to as the procedural knowledge needed in resolving a problem implies the ability to carry out basic computations and perform well-defined procedures. Lack of this knowledge poses a challenge in solving mathematical problems.

2. **Control Factors:** Control factors simply mean the ability to reflect, monitor and take decision on when and how to apply the various problem-solving knowledge and procedures. Successful problem-solving involves the control, monitoring, regulating and assessing of various knowledge and procedures utilized in solving problems otherwise known as metacognition. Kaur (1997) described metacognition as the ability to control one's own thinking processes in problem-solving and an important factor in "children's mathematical problem-solving". Thus, if a student lacks a control system to guide the selection of knowledge and procedures, he/she is bound to encounter difficulties in problem solving.
3. **Beliefs and Affective Factors:** The student's background belief about him/herself, nature of mathematics, topic and environment are influential to their behavior and disposition towards the learning of mathematical problem-

solving (Kaur, 1997). Also, affective factors such as unwillingness to learn, lack of perseverance and self-confidence also affect students' mathematical problem solving behavior. Thus, the negative belief system of students affects the attitude displayed when learning problem-solving and ability to solve problems effectively.

Conclusion

In life, every individual is confronted with one or more problems to tackle on a daily basis. This also is applicable to mathematics as a subject because students encounter problems as mathematical tasks that are expected to be resolved. Problem-solving is thus a major goal of mathematical teaching and learning as it enables learners to solve problems by establishing a relationship between the daily occurrence and the mathematical representation of such situations. George Polya's problem-solving process in tackling mathematics problems was examined. Despite this, students still find it difficult to learn mathematical problem-solving. This article expatiated the learning difficulties that can be encountered at each step of Polya's general process of problem-solving which include: lack of proper comprehension of the problem statement, lack of strategic competence, inability to implement an appropriate plan, and evaluation and verification difficulty. Factors contributing to these learning difficulties in problem-solving are also examined.

It is therefore recommended that teachers get acquainted with the various difficulties students can encounter in learning problem-solving and the factors that contribute to such. This will enable them devise means of helping students overcome these challenges so as to make them good problem solvers.

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