



Theoretical and Empirical Comparison of Stratified and Cluster Sampling Schemes with Application to Mathematics Students Scores

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Abstract

In this paper, we carried out a theoretical and empirical comparison of stratified and cluster sampling. Estimates such as variance of stratified sampling, variance of proportional allocation due to stratification, variance of optimal allocation due to stratification, and variance of cluster sampling were used for the comparison. Two data sets consisting of Mathematics scores of students in College of Education Akamkpa, were employed for the empirical validation of the theoretical comparison and the result shows that the stratified sampling variances were smaller than the cluster sampling variances in both cases. Hence stratified sampling results in gain due to stratification and therefore adjudged to be more efficient in relation to cluster sampling. The paper concluded that the performances of stratified sampling over cluster sampling is mainly dependent on the structure of the population, and to enhance efficiency of cluster sampling, there should be many clusters with few elements in it.

Keywords: Cluster sampling, Efficiency, Empirical comparison, Stratified sampling, Mathematics scores, and Variance.

Introduction

When a population cannot be observed by method of complete enumeration or census procedure due to financial inadequacy, limited time, manpower, scarcity of units etc., and we resort to sampling methods where a sample is drawn from the population for the purpose of making generalization or inferences about the population based on sample observation, the method of sampling always give rise to sampling error since only a portion of the population is observed. Another pause for thought is that neither complete enumeration nor sampling gives an exact result. To minimize error in surveys situation, it became imperative that we adopt a sampling method that is capable of giving an improved results and which will be adjudged to be a better representation of the population. Most often, interest has been on Simple Random Sampling (SRS) as a cost effective scheme for sample selection in most researches in the field of Sciences, Social Sciences, and surveys of natural resources. But SRS method usually suffers weak or loss of efficiency in the presence of high variation of data sets. Also, the abundance of new sampling techniques has further complicated the situation in the sense that a poor choice of a sampling method can lead to low precision, poor accuracy and unnecessary expenditure of time and money. To minimize this bad samples, possibility shall be concentrated on employing Stratified Simple Random Sampling (SSRS) and Cluster Sampling (CS).

In stratified sampling the population is first divided into subpopulations (strata) of smaller units which are non-overlapping and together they comprise the

whole population. Whereas cluster sampling entails a technique in which the population is divided into already existing natural, but heterogeneous intact grouping of members of the population. Literatures on sampling is quite vast and traceable to the early part of the 20th century with a sketch of history of survey sampling by Kovalevsky (1924), Bowley (1926) laid the foundational stone of modern sampling dealing with stratification, Neyman (1934, 1938), advocated two-phase sampling to address the problem of strata sizes in stratified sampling. For more studies on comparison of sampling techniques; Aneesh, S. Hariharan, V., Gallucci, F., & Graig, H., (2013) carried out an estimation of relative efficiency of adaptive cluster versus traditional Sampling designs with application to arrivals of sharks, Habib (2014) carried out a comparison of precision of systematic sampling with other probability Sampling, Qualite (2008) studied the comparison between conditional Poisson sampling versus unequal probability with replacement. Megan et al (2008) investigated precision, time, and cost as a comparison of three sampling designs in an emergency setting. Aires (1988) conducted a study on comparisons between conditional poisson sampling and parato πps sampling designs, Rose, A.M., Graig, R. F., Coulombier D., & Ritter, H. (2006) also carried a comparison of cluster and systematic sampling for measuring crude mortality. Shahrokh et al (2014) studied the effect of separate sampling on classification accuracy, while Xiaolu, Cao (2014) conducted a comparative study between Stratified random sampling and Pareto πps sampling.

In view to provide refinement to SRS and increased assurances that collected sample data will be a more representative of the entire population by using prior information to first divide the population into more similar subgroups and then employing the random sample approach within each subgroup called stratum to ensure broad representation across the whole population, this paper puts forward a theoretical and empirical comparison of stratified and cluster sampling with application to Mathematics students' scores in College of Education, Akamkpa. The paper discusses the efficiency of stratified sampling and cluster sampling using some theoretical and empirical evidences from a finite population analysis and recommend preferable situations for stratified sampling over clusters sampling vise-versa and how sampling errors can be reduced in stratified sampling and in cluster sapling.

Sampling Methods

Stratified random sampling description

In stratified sampling the population of N units is first divided into subpopulations of $N_1, N_2, N_3, \dots, N_L$ units respectively. These subpopulations are overlapping, and together they comprise the whole of the population, so that $N_1 + N_2 + N_3 + \dots + N_L = N$. The subpopulations are called strata. To obtain the full benefit from stratification, the values of N_L must be known. When the strata have been determined, a sample is drawn from each, the drawings being made independently in different strata. The strata size within the strata are denoted by $n_1, n_2, n_3, \dots, n_L$ respectively. If simple random sample is taken in each stratum, the whole procedure is described as stratified random sampling.

Notations

Let the number of strata = L for the h^{th} stratum (where $h = 1, 2, 3, \dots, L$)

The following symbols all refers to stratum h

Population size = N_h

Sample size = n_h

Stratum weight = $W_h = \frac{n_h}{N_h}$

Population size of all strata = $N = \sum_{i=1}^L N_h$ (1)

Sample size of all strata = $n = \sum_{i=1}^L n_h$ (2)

Population Mean $\bar{Y}_h = \sum_{i=1}^{N_h} \frac{y_{hj}}{N_h}$ (3)

Sample Mean $\bar{y}_h = \sum_{i=1}^{n_h} \frac{y_{hj}}{n_h}$ (4)

Population Variance = $S_h^2 = \frac{\sum_{j=1}^{N_h} (y_{hj} - \bar{Y}_h)^2}{N_h - 1}$ (5)

Sample variance $s_h^2 = \frac{\sum_{j=1}^{n_h} (y_{hj} - \bar{y}_h)^2}{n_h - 1}$ (6)

The proportion of the total population in stratum h

$\bar{Y} = \sum_{h=1}^L \sum_{i=1}^{N_h} \frac{y_{hj}}{N} = \sum_{i=1}^L N_h \frac{y_{hj}}{N} = \sum_{h=1}^L W_h \bar{Y}_h$ (7)

Sample proportion in stratum $h = p_h$

Population proportion in stratum $h = P_h$

The proportion of total population in stratum $h =$

$P_{str} = \sum_{h=1}^L W_h P_h$ (8)

$Var(P_{str}) = \sum_{h=1}^L W_h^2 \frac{(1-f_h) N_h P_h Q_h}{n_h} = \sum_{h=1}^L W_h^2 (1-f_h) \frac{p_h q_h}{n_h - 1}$ (9)

Since for proportional allocation $\frac{n_h}{n} = \frac{N_h}{N}$ or $f_h = f$

Variance of optimal allocation for stratification

$V_{opt}(\bar{y}_{str}) = \frac{\sum_{h=1}^L (W_h S_h)^2}{n} = \frac{S_0^2}{n}$ (10)

where $S_0 = \sum_{h=1}^L W_h S_h$

Variance of stratified random sampling $Var(\bar{y}_{str}) =$

$\sum_{h=1}^L W_h^2 (1-f_h) \frac{S_h^2}{n_h}$ (11)

Table 1 shows estimate of the population mean and variance per stratum for stratified sampling

Table 1: Description of Mean and variance per stratum

Stratum	1	2	3	.	.	.	L
N_h	N_1	N_2	N_3	.	.	.	N
n_h	n_1	n_2	n_3	.	.	.	n
W_h	$\frac{N_1}{N}$	$\frac{N_2}{N}$	$\frac{N_3}{N}$.	.	.	$\frac{N_h}{N}$
f_h	$f_1 = \frac{n_1}{N_1}$	$f_2 = \frac{n_2}{N_2}$	$f_3 = \frac{n_3}{N_3}$.	.	.	$f_n = \frac{n}{N}$
\bar{y}_h	$\frac{\sum y_1}{n_1}$	$\frac{\sum y_2}{n_2}$	$\frac{\sum y_3}{n_3}$.	.	.	$\sum_{i=1}^L \frac{y_{hj}}{n_h}$
S_h^2	$W_1^2 (1-f_1) \frac{S_1^2}{n_1}$	$W_2^2 (1-f_2) \frac{S_2^2}{n_2}$	$W_3^2 (1-f_3) \frac{S_3^2}{n_3}$.	.	.	$\sum_{h=1}^L W_h^2 (1-f_h) \frac{S_h^2}{n_h}$

Cluster Sampling Description

In cluster sampling, we assume that the population contains N clusters with M elements each, n cluster are selected from N clusters by simple random sampling without replacement (WOR) and all the elements in the selected clusters are included in the sample.

Notations

Let us denote Y_{ij} as the value of the characteristics under study for the j^{th} element in the i^{th} cluster

$$Y_{ij} = (j = 1, 2, 3, \dots, M, i = 1, 2, 3, \dots, N)$$

The i^{th} cluster mean or per element in the i^{th} cluster

$$\bar{Y}_i = \frac{\sum_{j=1}^M Y_{ij}}{M} \tag{12}$$

The mean of cluster mean $\bar{Y} = \frac{\sum_{i=1}^N \sum_{j=1}^M Y_{ij}}{MN}$ (13)

The mean square between elements in the i^{th} cluster

$$S_b^2 = \frac{\sum_{j=1}^M (Y_{ij} - \bar{Y}_i)^2}{M-1} \tag{14}$$

The mean square between the j^{th} clusters mean

$$S_B^2 = \frac{\sum_{i=1}^N (Y_{ij} - \bar{Y})^2}{N-1} \tag{15}$$

The mean square within the j^{th} cluster mean $S_W^2 =$

$$\frac{\sum_{i=1}^N \sum_{j=1}^M (Y_{ij} - \bar{Y})^2}{N(M-1)} \tag{16}$$

Estimate of the population mean $\bar{Y} = \frac{\sum_{i=1}^n \bar{y}}{n}$

$$\frac{\sum_{i=1}^n \sum_{j=1}^m Y_{ij}}{nm} \tag{17}$$

Variance of the population mean in cluster sampling

$$Var(\bar{Y}) = \frac{1}{nm} \left(1 - \frac{n}{N}\right) S_b^2 \tag{18}$$

Variance of cluster mean in terms of intra-cluster correlation coefficient $Var(\bar{Y}) =$

$$\frac{\frac{1}{n}(NM-1)S_y^2[1+(M-1)\rho]}{M^2[N-1]} \tag{19}$$

where $\rho = \frac{\sum_{i=1}^N \sum_{i \neq j}^M (Y_{ij} - \bar{Y})(Y_{ij} - \bar{Y})}{(NM-1)(M-1)S_y^2}$

Table 3 shows analysis of variance techniques for cluster sampling

Table 3: Analysis of variance techniques for cluster sampling

Sources of variation	SS	DF	MS
Between cluster	$M \sum (Y_i - \bar{Y})^2$	$N - 1$	$\frac{M \sum (Y_i - \bar{Y})^2}{N-1} = MS_B^2$
Within cluster	$\sum \sum (Y_i - \bar{Y})^2$	$N(M - 1)$	$\frac{\sum \sum (Y_i - \bar{Y})^2}{N(M-1)} = S_W^2$
Total		$MN - 1$	$MS_B^2 + S_W^2 = S_y^2$

When M elements in any cluster can be classified into two classes and that $P_i = \frac{a_i}{M}$ is the proportion in class C in the i^{th} cluster. A simple random sample of n cluster is taken, and the average P of the observed P_i is the sample used as the estimate of the population P .

Then variance of cluster sampling for proportion is given as $Var(P) = \frac{N-n}{Nn} \frac{\sum_{i=1}^N (P_i - P)^2}{N-1} = \frac{N-n}{N^2n} \sum (P_i - P)^2$ (20)

Theoretical Comparison in Terms of Precision and Relative Precision

Precision of stratified random sampling over cluster sampling

For precision of stratified random sampling over cluster sampling,

let $V(\bar{y}_{cls}) = \frac{(N-1)s_b^2 + N(M-1)s_w^2}{NM-1}$ be the variance of cluster sampling, and let

$$V_{Prop}(\bar{y}_{str}) = \sum_{h=1}^L w_h^2 (1 - f_h) \frac{N_h P_h Q_h}{N_h - 1},$$

$V_{opt}(\bar{y}_{str}) = \sum_{h=1}^L \frac{(w_h s_h)^2}{n}$, and $V(\bar{y}_{str}) = \sum_{h=1}^L w_h^2 (1 - f_h) \frac{s_h^2}{n_h}$, be the variance for proportion, optimal, and variance due to stratification in stratified random sampling, then the comparison of stratified random sampling results in gain due to stratification if:

$$V_{opt}(\bar{y}_{str}) \leq V_{Prop}(\bar{y}_{str}) \leq V(\bar{y}_{cls})$$

$$a \leq b \leq c$$

$$\Rightarrow a \leq c \text{ by transitivity}$$

$$V_{opt}(\bar{y}_{str}) \leq V(\bar{y}_{cls})$$

$$\sum_{h=1}^L \frac{(w_h s_h)^2}{n} \leq \frac{(N-1)s_b^2 + N(M-1)s_w^2}{NM-1}$$

$$\sum_{h=1}^L \left(\frac{w_h s_h \sqrt{n}}{n}\right)^2 \leq S_y^2$$

$$\sum_{h=1}^L \left(\frac{w_h s_h \sqrt{n}}{n}\right)^2 - S_y^2 \geq 0 \left(\sum_{h=1}^L \left(\frac{w_h s_h \sqrt{n}}{n} + S_y\right) * \left(\sum_{h=1}^L \left(\frac{w_h s_h \sqrt{n}}{n} - S_y\right)\right) \geq 0\right)$$

$$(21)$$



where $S_y^2 = \frac{(N-1)s_b^2 + N(M-1)s_w^2}{NM-1}$ and $S_y = \sqrt{\frac{(N-1)s_b^2 + N(M-1)s_w^2}{NM-1}}$

Remark 1: Since the population size N and the sample n drawn from it are always greater than zero (0), the condition in (21) holds implying that stratified sampling outperforms cluster sampling. The variance of optimum allocation due to stratification is smaller than the variance of cluster sampling. The comparison of stratified sampling in relation to cluster sampling results in gain due to stratification.

Relative precision of stratified sampling over cluster sampling

The relative precision or efficiency of stratified sampling over cluster sampling is expressed as

$$R * E(\bar{y}) = \frac{V(\bar{y}_{str})}{V(\bar{y}_{cls})} < 1$$

i.e $R * E(\bar{y}) = \frac{\sum_{h=1}^L w_h^2 (1-f_h) \frac{s_h^2}{n_h}}{\frac{(N-1)s_b^2 + N(M-1)s_w^2}{NM-1}} < 1$

(22)

Whenever is convenient or without loss of generality, we can estimate the percent relative efficiency $P.R.E$ as $P.R * E(\bar{y}) = R * E(\bar{y}) * 100$

i.e $\frac{V(\bar{y}_{str})}{V(\bar{y}_{cls})} * 100 < 100$

$$P.R * E(\bar{y}) = \left(\frac{\sum_{h=1}^L w_h^2 (1-f_h) \frac{s_h^2}{n_h}}{\frac{(N-1)s_b^2 + N(M-1)s_w^2}{NM-1}} \right) * 100 < 100$$

(23)

Remark 2: For stratified sampling to outperform cluster sampling, the result in (22) must be less than

one (1), i.e $(R * E(\bar{y})) < 1$, and the result in (23) must be less than 100, i.e $(P.R * E(\bar{y})) < 100$. However, if $R * E(\bar{y}) = 1$ or $P.R * E(\bar{y}) = 100$, then stratified sampling is the same as cluster sampling. It could be recalled that an estimator is said be efficient over its competitor’s estimator if the estimator in question has the smallest variance. If the inverse of the conditions in (21), (22), and (23) holds, then cluster sampling is efficient in relation to stratified sampling. Efficiency of cluster sampling would be enhanced if we have many clusters with few elements in it.

Empirical study

In order to validate the theoretical results and to compare the efficiency of the stratified sampling over cluster sampling, empirical study was conducted using data obtained from MTH 322 (Linear Algebra) results of students in Mathematics Department of Cross River State College of Education, Akamkpa, with the following course combinations Maths/Chem, Maths/Econs, Maths/Geo, Maths/Phy, Maths/ISC, between 2015/2016 to 2016/2017 academic session.

Presentation of Data

We represent each course combination in the Mathematics Department by a Stratum, i.e Maths/Chem, Maths/Econs, Maths/Geo, Maths/Phy, and Maths/ISC shall denote stratum1= (y_1) , stratum2 = (y_2) , stratum3= (y_3) , stratum4= (y_4) , stratum5 = (y_5) respectively. Randomly selected samples gave the data in Table 3 which constitute of MTH 322 results for 2015/2016 academic session while Table 4 shows data of MTH 322 results for 2016/2017 academic session.

Table 3: Sample of MTH 322 results for 2015/2016 academic Session

Stratum 1	Stratum 2	Stratum 3	Stratum 4	Stratum 5
47	32	33	80	68
35	68	41	50	53
40	60	26	34	45
45	40	55	28	34
66	65	22	43	53
42	40	37	38	44
42	50	32	72	40
42	43	42	44	31
42	44	57	48	33
42	42	40	54	49

Table 4: Sample of MTH 322 results for 2016/2017 academic Session

Stratum 1	Stratum 1	Stratum 3	Stratum 4	Stratum 5
45	15	41	59	52
40	60	31	27	45
28	45	40	52	47
51	25	45	45	25
58	50	24	62	32



Table 5: Analysis of MTH 322 results for 2015/2016 session

	Stratum 1	Stratum 2	Stratum 3	Stratum 4	Stratum 5	Row Total	(Row Total) ²
	47	32	33	80	68	260	67600
	35	68	41	50	53	247	61009
	40	60	26	34	45	205	42025
	45	40	55	28	34	202	40804
	66	65	22	43	53	249	62001
	42	40	37	38	44	201	40401
	42	50	32	72	40	236	55696
	42	43	42	44	31	202	40804
	42	44	57	48	33	224	50176
	42	42	40	54	49	227	51529
Column Total	443	484	385	491	450		512045
(Column Total)²	196249	234256	148225	241081	202500	1022311	

Table 6: Analysis of variance (ANOVA) for 2015/2016 Session

Sources of variation	D.F	S.S	M.S
Between Strata	9	444.41	49.38 = S_b^2
Within Strata	40	51470.91	1286.77 = S_w^2
Total	49	58040.91	1336.15* = $V(\bar{y}_{cls}) = S_b^2 + S_w^2$

Table 7: Analysis of mean and variance per stratum for 2015/2016 session

	y_1	$(y_1 - \bar{y}_1)^2$	y_2	$(y_2 - \bar{y}_2)^2$	y_3	$(y_3 - \bar{y}_3)^2$	y_4	$(y_4 - \bar{y}_4)^2$	y_5	$(y_5 - \bar{y}_5)^2$
	47	7.29	32	268.96	33	30.25	80	954.81	68	529
	35	86.49	68	384.16	41	6.25	50	0.81	53	64
	40	18.49	60	134.56	26	156.25	34	228.01	45	0
	45	0.49	40	70.56	55	272.25	28	445.21	34	121
	66	470.89	65	275.56	22	272.25	43	37.21	53	64
	42	5.29	40	70.56	37	2.25	38	123.21	44	1
	42	5.29	50	2.56	32	42.25	72	524.41	40	25
	42	5.29	43	29.16	42	12.25	44	26.01	31	196
	42	5.29	44	19.36	57	342.25	48	1.21	33	144
	42	5.29	42	40.96	40	2.25	54	24.01	49	16
total	443	610.1	484	1296.4	385	1138.5	491	2364.9	450	1160
\bar{y}_h	44.3		48.4		38.5		49.1		45	
S_h^2		67.789		144.044		126.5		262.767		128.889

Table 8: Estimates of population mean and variance of stratification for 2015/2016 session

Stratum	1	2	3	4	5	
N_h	35	40	25	30	20	N=150
n_h	10	10	10	10	10	n=50
\bar{y}_h	44.3	48.4	38.5	49.1	45	
w_h	0.233333333	0.266666667	0.166666667	0.2	0.133333333	
f_h	0.285714286	0.25	0.4	0.333333333	0.5	
s_h^2	67.78889	144.0444	126.5	262.7667	128.8889	
\bar{y}_{str}	10.33666667	12.90666667	6.416666667	9.82	6	45.48
$Var(\bar{y}_{str})$	0.263623461	0.7682368	0.210833333	0.7007112	0.114567911	2.057973*



Table 9: Analysis of MTH 322 results for 2016/2017 session

	Stratum 1	Stratum 1	Stratum 3	Stratum 4	Stratum 5	Row Total	(Row Total) ²
	45	15	41	59	52	212	44944
	40	60	31	27	45	203	41209
	28	45	40	52	47	212	44944
	51	25	45	45	25	191	36481
	58	50	24	62	32	226	51076
Column Total	222	195	181	245	201		218654
(Column Total) ²	49284	38025	32761	60025	40401	220496	

Table 10: Analysis of variance (ANOVA) for 2016/2017 session

Sources of variation	D.F	S.S	M.S
Between Strata	4	133.36	33.40 = S_b^2
Within Strata	20	501.76	25.08 = S_w^2
Total	24	3964.56	58.49* = $V(\bar{y}_{cls}) = S_b^2 + S_w^2$

Table 11: Analysis of means and variance per stratum for 2016/2017 session

	y_1	$(y_1 - \bar{y}_1)^2$	y_2	$(y_2 - \bar{y}_2)^2$	y_3	$(y_3 - \bar{y}_3)^2$	y_4	$(y_4 - \bar{y}_4)^2$	y_5	$(y_5 - \bar{y}_5)^2$
	45	0.36	15	576	41	23.04	59	100	52	144
	40	19.36	60	441	31	27.04	27	484	45	25
	28	268.96	45	36	40	14.44	52	9	47	49
	51	43.56	25	196	45	77.44	45	16	25	225
	58	184.96	50	121	24	148.84	62	169	32	64
Total	222	517.2	195	1370	181	290.8	245	778	201	507
\bar{y}_h	44.4		39		36.2		49		40	
s_h^2		129.3		342.5		72.7		194.5		126.75

Table 12: Estimates of population mean and variance of stratification for 2016/2017 session

Stratum	1	2	3	4	5	Total
N_h	20	35	20	15	10	N=100
n_h	5	5	5	5	5	n=25
\bar{y}_h	44.4	39.00	36.2	49.00	40.00	
w_h	0.20	0.35	0.20	0.15	0.1	
f_h	0.25	0.14	0.25	0.33	0.5	
s_h^2	129.3	342.5	72.7	194.5	126.75	
\bar{y}_{str}	8.88	13.65	7.24	16.17	6	41.12
$Var(\bar{y}_{str})$	0.7758	7.216475	0.4362	0.586418	0.12675	9.1416245

Table 13: Summary of results of variances of the sampling methods and their relative and percent relative efficiency

Variance	2015/2016	2016/2017	$R.E(\bar{y})$	$P.R.E(\bar{y})$
$V(\bar{y}_{cls})$	1336.15	58.49	0.00154 < 1	0.154
$Var(\bar{y}_{str})$	2.057973	9.1416245	0.156294 < 1	15.63

Conclusion

The empirical study of this paper considered two approaches namely: Relative Efficiency ($R.E(\bar{y})$) and Percent Relative Efficiency ($P.R.E(\bar{y})$) to ascertain the veracity of theoretical underpinnings

which was demonstrated and shown in (22) and (23) respectively, from where it was observed that stratified sampling outperform cluster sampling, and results in gain due to stratification, since the result in (22) is less than one (1), i.e ($R * E(\bar{y}) < 1$), and the

result in (23) is less than 100, i.e $(P.R * E(\bar{y})) < 100$). However, if $R * E(\bar{y}) = 1$ or $P.R * E(\bar{y}) = 100$, then stratified sampling is the same as cluster sampling. This results is similar to that of the work of R.F Calombier D. Ritter H. (2006) comparison of two different survey methods, and Xiaolu Cao (2014) comparison between stratified random sampling and Pareto πps sampling.

In order to validate the theoretical results, we computed and compared the efficiency of the stratified sampling over cluster sampling, empirical results were employed using data set from students MTH 322 (Linear Algebra) results shown in table 3 and table 4 respectively. Analysis of variance (ANOVA) for the data was carried out to compute the within group and between group variances, their sum gave the cluster sampling variance. Analysis of mean and variance per stratum, analysis of estimates of population mean and variance of stratification, summary of results of variance of the sampling methods and their relative efficiencies for the two sets of data were carried out and the results presented in table 5, table 6, table 7, table 8, table 9, table 10, table 11, table 12 and table 13 respectively. The results from the analysis revealed $V(\bar{y}_{str}) < V(\bar{y}_{cls})$ i.e $(2.057973) < (1336.15)$ for case I and $V(\bar{y}_{str}) < V(\bar{y}_{cls})$ i.e $9.1416245 < 58.49$ for case II respectively. Also, Relative Efficiency $(R.E(\bar{y}))$ of $(0.00154), (0.156294)$ and Percent Relative Efficiency $(P.R.E(\bar{y}))$ of $(0.154), (15.63)$ were obtained for both cases. This result is akin to that Habib Ahmed Elsayir (2014) on comparison of stratified sampling versus systematic sampling.

Conclusively, for the two sets of data employed, it was observed that $V(\bar{y}_{str})$ was less than $V(\bar{y}_{cls})$, secondly, $R.E(\bar{y})$ was less than 1 and $P.R.E(\bar{y})$ was less than 100. Hence, stratified sampling is adjudged to be more efficient than cluster sampling. It could be recalled that an estimator is said to be efficient over its competitor's estimator if the estimator in question has the minimal variance. However, there may be population for which cluster sampling would be extremely precise or efficient in relation to stratified sampling and others for which it will be less efficient. Therefore, the performance of stratified sampling over cluster sampling is mainly dependent on the population in question.

Recommendation

Based on the study, we recommend thus:

1. As observed in the numerical application, social scientist, marketers, Agricultural scientist and other field workers should conduct their surveys taking into

consideration the advantage of stratified sampling in relation to cluster sampling.

2. Academic scholars (Statisticians) should desire the extension of this comparison to other probability sampling methods. E.g Simple random sampling, systematic sampling, and multistage sampling (in which some of the sampling methods above are combined in one stage or the other).
3. Researchers should apply the stratified sampling approach to research data audits, with focus on illustrating the use of a stratified sampling technique.
4. Application of sampling to agricultural statistics with emphasis on stratified samples. Data on agricultural items, in general, are extremely heterogeneous, and efficient sampling program to obtain information about agriculture almost invariably requires stratification. Here, we recommend that the theories of stratified sampling to be utilized in increasing the accuracy of agricultural data collected on a sample basis.

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