



The Use of Geogebra to Develop Mathematical Knowledge of Transformations in Functions and Graphs

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Abstract

This study investigated the utilitarian value of GeoGebra as a pedagogical tool for understanding and moving to higher levels of abstraction while learning transformation of functions and graphs in mathematics. This study adopts the qualitative paradigm to understand the processes involved in using GeoGebra. The population for this study were students enrolled in the bridge programme offered by a University in South Africa. The study involved a purposeful sample of students enrolled in the bridging program at Stellenbosch University. The instrument for the study were observation schedules and in-depth/focus group interviews. Observation data revealed that utilizing GeoGebra as an instructional tool allowed for a variety of instrumental orchestrations, which proved to be particularly beneficial in the educational setting. Additionally, the findings from in-depth interviews and focus group discussions provided valuable insights into the students' experiences. Participants expressed that the visual affordances of GeoGebra, along with its potential for interactive engagement, facilitated a deeper comprehension of complex concepts related to function and graph transformations. Furthermore, the study highlighted the importance of these visual affordances in helping students connect rules of transformations given to them to visual representations of the rules, ultimately improving their overall mathematical understanding. Based on the conclusion of the study, engaging a guided interaction with GeoGebra during lessons, encouraging student-centered GeoGebra projects that require students to create and analyze their own GeoGebra constructions in graphs, sketches, and dynamic visualizations and designing learning activities that explicitly link concrete manipulation are recommended.

Keywords: Technology-enhanced learning, GeoGebra, Transformation of functions and graphs, Mathematics

Introduction

Technology has changed the narrative in all walks of life and mathematics teaching is not an exemption. Mathematicians have commonly used calculators, and technological tools and applications to simplify mathematics (Pohl & Hornbuk, 2024). The 1980s saw the demonstration of the computer's potential for mathematical teaching. By integrating technology into mathematics education, numerous nations are currently transforming the way that mathematics is taught and learnt (Mehanovic, 2009). Ndlovu et al. (2011) also reference the fact that there is a growing need to integrate technology the into mathematics instructions. Technology is evolving, putting pressure on schools to incorporate it into the curriculum and becoming a permanent fixture in our lives, particularly in the era of the fourth industrial revolution (Oladele, 2024).

Since the 1960s, computers have been used to teach mathematics (Roberts, 2013). As a result, technology is increasingly being leveraged at all

levels of education to enhance mathematics teaching and learning processes (Attard & Holmes, 2020; Strutchens et al., 2019). This rapid proliferation of technology integrations in South African classrooms have significantly impacted classroom instruction. The emphasis on using available technology in the CAPS (DBE, 2011a) demonstrates that South Africa is not an exception to this excitement. South Africa like other sub-Saharan African nations continue to have relatively low levels of information and communication technology (ICT) readiness (Dutta et al., 2015; Oladele, 2024), emphasis must be laid on the using technology to teach mathematics. Despite widespread computer access among teachers and students and the availability of appropriate software in schools and homes, the daily teaching of mathematics rarely incorporates technology. Downloading and installing GeoGebra from the internet is free for teachers and students with computers or smartphones. GeoGebra can also

function as a web-based application, meaning you can access and use its features directly through your web browser without needing to download and install any separate software; it allows users to create interactive mathematical visualizations, graphs and geometric constructions online, making it accessible from any device with an internet connection.

The conventional teaching method has been reported as less effective in a technology-mediated environment since students can work at different speeds with varying cognitive capacities and this requires teachers to pay attention to learning styles and related classroom dynamics (Ndlovu, 2004; Ndlovu et al., 2011). Computer technology can strongly challenge the traditional teaching approach, and this necessitates a change in the roles of the teacher in teaching processes (Oladele et al., 2023).

Some advantages of using ICT for learning is its ease of feedback, flexibility, faster time which enhances experiential learning and the availability of open-source software which is relevant for solving funding constraints (Actaφ'eba et al., 2024; Guillén-Gámez et al., 2018; Oladele et al., 2023; Orji, 2024). Technology is an indispensable pedagogical tool for instilling mathematical knowledge. Therefore, educational institutions are deemed it important to provide students with educational required software packages for learning mathematics (Velychko et al., 2020). GeoGebra is a freeware and shareware that solves the problem of high cost usually associated with acquiring other technology-driven pedagogical applications (Tamam & Dasari, 2021).

GeoGebra aids visualisation of mathematics concepts through collaborative and hands-on engagements with the teacher and other students. This experience is enhanced in the virtual environment of the application which makes it applicable in distant learning programmes (Özgün-Koca & Meagher, 2012; Sattar et al., 2015). asserts that manipulatives function of GeoGebra further aids the learning while increasing students' interest in and enjoyment of mathematics (Moore, 2017; Miller et al., 2021; Serpa Howe, 2024).). Empirical evidence has shown that using the right technology to spark a sustained interest in mathematics has improved mathematical proficiency (Bright et al., 2024; Harefa, 2023). The PISA assessment revealed that students who are more interested in and like mathematics typically perform better academically than those who are less interested and enjoy the subject (OECD, 2004; Brozo, 2007). Furthermore, Cockett and Kilgour (2015) discovered that using manipulatives to teach mathematical concepts is beneficial. Additionally, they note that students believe that using arithmetic manipulatives

improves their understanding of the subject, increases their productivity, and makes studying more enjoyable. The manipulation functionality of the application is relevant as student engagement during mathematics lessons aids assimilation and learning (Attard, 2012). Students must be able to communicate and share their thoughts, as well as understand why mathematics functions in particular ways. As such, students' interest in mathematics enhances learning (Gullage, 2024). Therefore, teaching mathematics should also be enhanced by leveraging interaction with virtual reality technologies, being a catalyst for horizontal and vertical mathematisation, which aids memorising algorithms (Çakıroğlu et al., 2024). Furner & Marinas (2013) emphasised the need to introduce technology to enhance students' engagement during mathematics classes.

The Department of Education of South Africa introduced technology to improve pedagogical engagements. While access vary across school type and location, the provided resources are regarded as under-utilised. (DBE, 2011b). This may be alluded to South Africa's failure to close the "digital divide". The digital divide may be reduced if ICT is used in schools to boost learning. This would assist overcome some of the obstacles to increasing the effectiveness and productivity of teaching and learning in South African schools (Mdlongwa, 2012). Another challenge was ICT readiness in South Africa which was also reported as very low fewer teachers and students have access to computers (Dutta et al., 2015). Nkula and Krauss (2014) suggest that while many South African schools do have access to various ICT tools, their use is often limited. Nkula and Krauss (2014) assert that the lack of integration in the implementation of ICTs contrasts with the integration approach, where students use ICTs to enhance learning. de Jong et al. (2024) proposed that teachers can enhance their effectiveness by offering explicit guidance, practice, and feedback using relevant ICT resources for learning.

Theoretical and Conceptual Frameworks

This research uses instrumental genesis (TIG) and Piaget's cognitive constructivism as its main theoretical framework to investigate how students' progress to higher levels of abstraction in the transformation of functions and graphs.

TIG gives objects that support human action during task execution (Drijvers et al., 2013). Two processes—instrumentation and instrumentalization—combine to form instrumental genesis, according to Trouche (2004).

Instrumentation was defined as the process by which the user moulds the item in accordance with prior working methods and knowledge while developing activity schemes within boundaries that aided completing tasks in novel ways (Fahlgren, 2015; Ndlovu et al., 2011). These boundaries include enablements that effectively enable students to performing certain action, peculiar restrictions and potentialities that fundamentally open up options and affordances that favour onward movement sequences (Trouche, 2004). As such, instrumental genesis focuses on how students can engage the use of schemes that support the development and representation of concepts related to function transformation by leveraging GeoGebra as a solution-driven resource (Drijvers et al., 2013). According to this study, the TIG framework explains how GeoGebra was sets up to make a learning habitat for teaching function transformations.

Cognitive constructivism is believed to be an individualistic viewpoint as ingrained in Jean Piaget (1896-1980) views of human intellectual development. Three types of knowledge and the methods were identified through which students learn each one: social, physical, and logico-mathematical knowledge. Social knowledge is limited to interactions with other people and relates to customs that individuals have developed over time (Kamii, 2014; Lutz & Huitt, 2004). On the other hand, physical knowledge can be acquired through direct interaction with the learning tools or applications. This is applicable with GeoGebra for exploring concepts through which physical knowledge is developed, which serves as the foundation for logico-mathematical thinking (Piaget, 1952). Abstract thinking is a component of logico-mathematical knowledge that extends beyond tangible interactions with concrete stimuli and aided by real-time manipulation of objects which facilitates the formation of generalizations and abstractions (Lutz & Huitt, 2004).

The Problem

The traditional instructional methods are passive in nature and has been regarded as less effective for only a minority of learners with respect to mathematics and can become both boring and restrictive for students (Carless, 2022; Lokan et al. (2003). Verma et al. (2024) argued that in “this technologically advanced era it would seem natural to question whether relatively accessible and affordable technologies can contribute towards addressing the poor quality of teaching and learning”. However, there are sparing mathematics interventions that focus on the use of technology in

the teaching and learning of mathematics (Padayachee et al., 2011). Considering that the new crop of students are digital natives, teachers can employ technology to create an enjoyable and discovery-driven mathematics learning environment. Functions and graphs are one of the Grade 10–12 topics that were selected from the Curriculum Assessment Policy Statement (CAPS) (DBE, 2011a). Using GeoGebra as a stimulating technological tool, instructional activities can be driven through the discovery teaching method with time-saving benefits while exploring and investigating multiple activities (Akuu & Zengin, 2022; Seftiana et al., 2024). This design promotes the activity and guided reinvention tenets of realistic mathematics education (RME) are encouraged by this approach. By showing students how realistic gains behave worldwide, GeoGebra aids in the development of a structural knowledge of function. Against this background, this study examined the feasibility of teaching and learning how to convert functions and graphs using the dynamic mathematics program GeoGebra as guided by the following research questions:

1. How is GeoGebra used as a teaching tool to aid comprehension of transformation of functions and graphs?
2. How did GeoGebra help students grasp mathematical ideas and advance to more abstract levels?

Research Methodology

Design

This study adopted qualitative research paradigm for understanding how the use of GeoGebra helped students to move to higher levels of abstraction while learning transformation of functions and graphs. In qualitative research, the naturalistic approach seeks to comprehend phenomena in particular contexts, such as the real world, without attempting to manipulate the phenomenon of interest in order to support the hypotheses, explanations, and conceptualisations that were developed using participant data (Harwell, 2011). Therefore, quantitative research could value voices, thoughts, and perspectives.

Population and Sampling

48 students that were enrolled in the SciMathUS bridge program offered in a South African made up the study's population. The program afforded students who are educationally disadvantaged (economically and location wise) another opportunity to be eligible for admission to STEM programs at the study institution or any other

of their choosing. The programme was designed to enhance students Grade 12 grades during this year-long program with the option of mathematics with accounting and mathematics with physical sciences. Additionally, Thinking and Studying Skills, Academic Literacy and Language, and Computer Literacy are also available. However, only the mathematics and physical science portions of the National Senior Certificate Examination are retaken by the students. Students are encouraged to develop their critical thinking skills through SciMathUS's comprehensive curriculum approach with clearly stipulated admission requirements (Malan et al., 2014). Of the 48 SciMathUS students who participated in this study, 39 (81.3%) scored lower than 60% on their prior NSC maths exams, making up an identical portion of the sample for the study. The population for the study was sampled using the convenient sample technique which was premised on their availability or expediency (Sherman, 2024). From this group, students were further selected for the focus groups and in-depth interviews using basic random sampling technique (Rahman et al., 2022).

Instrumentation

The data for this study was collected using observation and in-dept and focus group interviews.

Data Analysis Techniques

The study's qualitative data analysis concentrated on the students' obstacles and enjoyable experiences while learning using GeoGebra. The data collected were transcribed which informed the coded and themes generated in ATLAS.ti. Figure 1 shows the coding of students' responses from in-depth interviews.

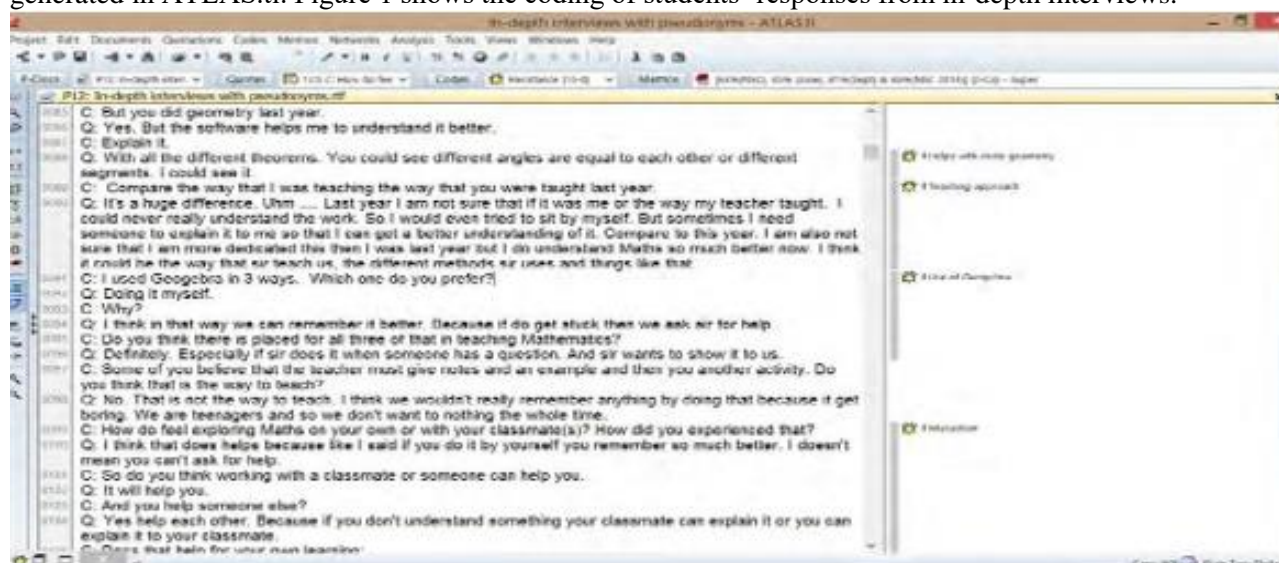


Figure 1: The coding process using ATLAS.ti

The classroom observations: The researcher fulfilled dual roles as both a direct observer and a participant observer throughout this study. As dedicated teacher-researcher, they closely monitored and recorded the students as they engaged with GeoGebra to explore a variety of graph transformations. During this process and the detailed analysis of the recording, the researcher documented students' behaviours, interactions, and the different types of instrumental orchestrations they employed.

The interview schedule: these instruments were semi-structured and designed for conducting the in-depth and focus group interviews with participants after they were taught using GeoGebra. Simple random sampling was used to select a sample of 10 students, with 5 students chosen from each class for the in-depth interviews with their full consent. The recorded interview sessions were transcribed and analysed using ATLAS.ti. Furthermore, 25 students were chosen at random for the focus group interviews out of the 38 who did not take part in the in-depth interviews. There were three focus groups with six members each, and one group with seven, for a total of twenty-five. 23 students participated in four focus group interviews after two students chose not to participate.

Interaction, enjoyment, visualisation, challenges, real-life, resistance, were some of the themes that emerged from the analysed data from the in-depth interviews. Figure 2 shows the summary of the code manager in ATLAS.ti.

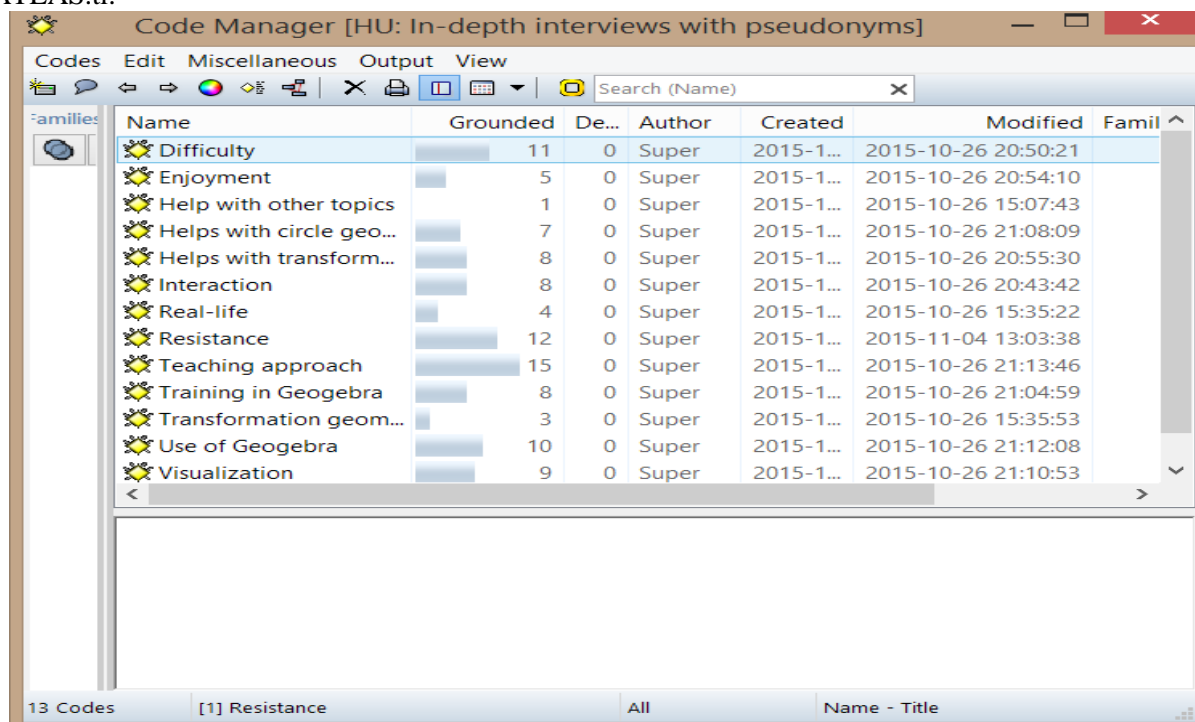


Figure 2: Summary of themes in ATLAS.ti code manager

Ethical clearance was obtained from the University's Research Ethics Committee (REC) to conduct the research with the students enrolled for the bridge programme. Ethical measures for the study were informed consent where all participants were fully

informed about the purpose of the study and procedures. Confidentiality was also ensured with data anonymized to protect participants' identities and voluntary participation was ensured with no penalty in the case of withdrawal.

Result

Research Question 1: How is GeoGebra used as a teaching tool to aid comprehension of transformation of functions and graphs?

The researcher examined the various instrumental orchestrations used by the teacher-researcher and examined the video footage of the classroom observations. These instrumental orchestrations describe the interaction in terms of the steps involved in using GeoGebra as a teaching tool using the musical approach as shown in Table 1.

Table 1: Frequency counts of orchestrations during the transformations of functions instruction

Orchestration type	First 50 minutes computer lab (1 st session): Frequency	Next 50 minutes of computer lab (2 nd session): Frequency	Next 100 minutes of computer lab (3 rd & 4 th session): Frequency	Next 100 minutes of computer lab (5 th & 6 th session): Frequency
Technical demo	3	2	2	1
Explain-the-screen	4	5	2	1
Link-the-screen	5	3	2	2
Discuss-the-screen	6	3	2	2
Guide-and-explain	Most of the time	Most of the time	Most of the time	Most of the time

All five orchestration types used are displayed in the first two sessions. At first, it was crucial to address technical concerns and have class-wide conversations and explanations to help kids feel at ease using computers. Additionally, it was critical that the students learn how to study the mathematics and become acquainted with the program. The researcher described how GeoGebra's installation and portable versions operate during the technical demo orchestrations while emphasising the interface and functioning of the key icons with the required technical support.

Table 1 also shows how the *technical demo*, *Explain-the-screen*, *Link-the-screen* and *Discuss-the-screen* orchestrations were less frequently used in sessions three to six. The researcher provided more frequent computer help to the students, i.e., the orchestration of guidance and explanation. Students in this orchestration received assistance when they needed it with computer-related tasks. Most of the time, students followed directions from the researcher which encouraged individualised and experiential learning. This activity is congruent with cognitive and radical constructivism for students to construct knowledge on their own.

This result showed that all orchestrations were relevant pedagogical tool when using GeoGebra for understanding the transformation of functions while putting support mechanisms that are crucial for effective engagements.

Research Question 2: How did GeoGebra help students grasp mathematical ideas and advance to more abstract levels?

The subsequent section examines the students' answers from the focus group (FG) and in-depth (ID) interviews following their initial computer lab instruction of GeoGebra and their initial instruction utilising a combination of an active learning approach and a teacher acting as a learning facilitator.

Student ID1: In the mother graph, I could observe [visual affordances] and observe the changes that take place in real time. I could also see when the formula changes [potentialities], resulting into transformations. Similarly, Student ID6 affirmed that it was quite beneficial. The meaning of a (parameter) in transformations was unknown to me. The responded indicated that seeing it aids remembering which connotes visual affordances.

Student FG9: it was initially highly puzzling like $(x + 30^\circ)$ but using GeoGebra made it

clearer as plus 30° . This confusion led to adding 30 and then move that to the right, but $x + 30^\circ$ should moving to the left to be correct. There will then be a change, and you can actually see how it changed [visual affordances]. This skill was said to be very helpful in solving exercises related to graphing.

Student AT1: I was able to appreciate the function's graph through the pictorial functions [visual affordances] which aided my understanding of how the graph's equation got altered using GeoGebra [enablement].

Student AT2: It was interesting to use the GeoGebra app on the computer to shift graphs since it provided a better image of the changes made to a graph, which makes it easier to remember information. Viewing the graphs in various shades [enablement] made them easier to compare with the original and also aid simplification of the learning concepts.

Students discussed how they were able to observe that the graph displayed a form of

transformation when the equation's parameters changed. This implies that GeoGebra aids students' progression from a peripheral to a deeper level of comprehension (Sattar et al., 2015).

When asked how GeoGebra aided in their understanding of transformation, the students responded by providing the following specific examples of how GeoGebra improved their comprehension of function transformations:

Student ID9: I was aware last year that the x -value would shift to either a positive or negative number if you thought about the y -axis. This was a rote way of learning, but with GeoGebra learning becomes more easier as aided by sight [visual affordance], and this aided learning.

These findings show that majority of students gave answers in favour of GeoGebra for improving their comprehension of ideas related to transformations of functions and graphs.

Figure 3 portrays the reflection of a point in the y -axis as aided by the use of GeoGebra.

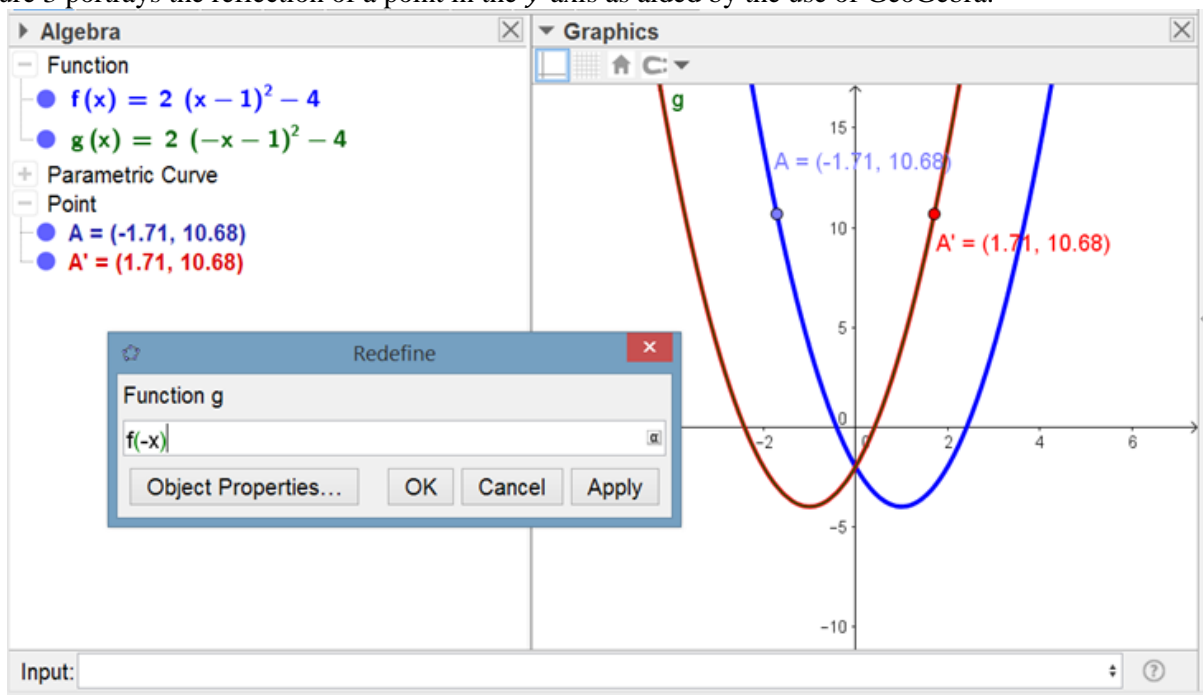


Figure 3: GeoGebra applet for teaching reflection in y -axis

Student ID9 talked about how she could see that point A is mapped to A' if it were mirrored in the y -axis. The student pointed out that the only way to get the coordinates of A' is to make the x -value positive. As a result, the student was able to develop logico-mathematical knowledge by learning the rule for reflection in the y -axis using GeoGebra and

grasping the concept of negating the x -value. Through explaining transformations of functions with input and output values, the student now understood why the rule for the reflection in the y -axis is: $(x; y) \rightarrow (-x; y)$ (Faulkenberry & Faulkenberry, 2010)

Responding to the researcher's question on if GeoGebra aided algebraic notation of the transformation of functions are the following:

Student FG16: To be honest, I didn't enjoy the transformation's dilation phase, but the shifting and reflection phases were incredibly beneficial, particularly for the exponential graph..... If In the absence of visuals, the mirror image of it can be

drawn [enablement], this activity also aided learning of the topic a great deal.

Student FG16 expressed how she used only points to draw the exponential function's inverse graph. Instruction on how to sketch the graphs of the altered functions using dots or critical spots helped with this. Figure 4 demonstrates how students were introduced to using GeoGebra to learn about the inverse graph of exponential functions.

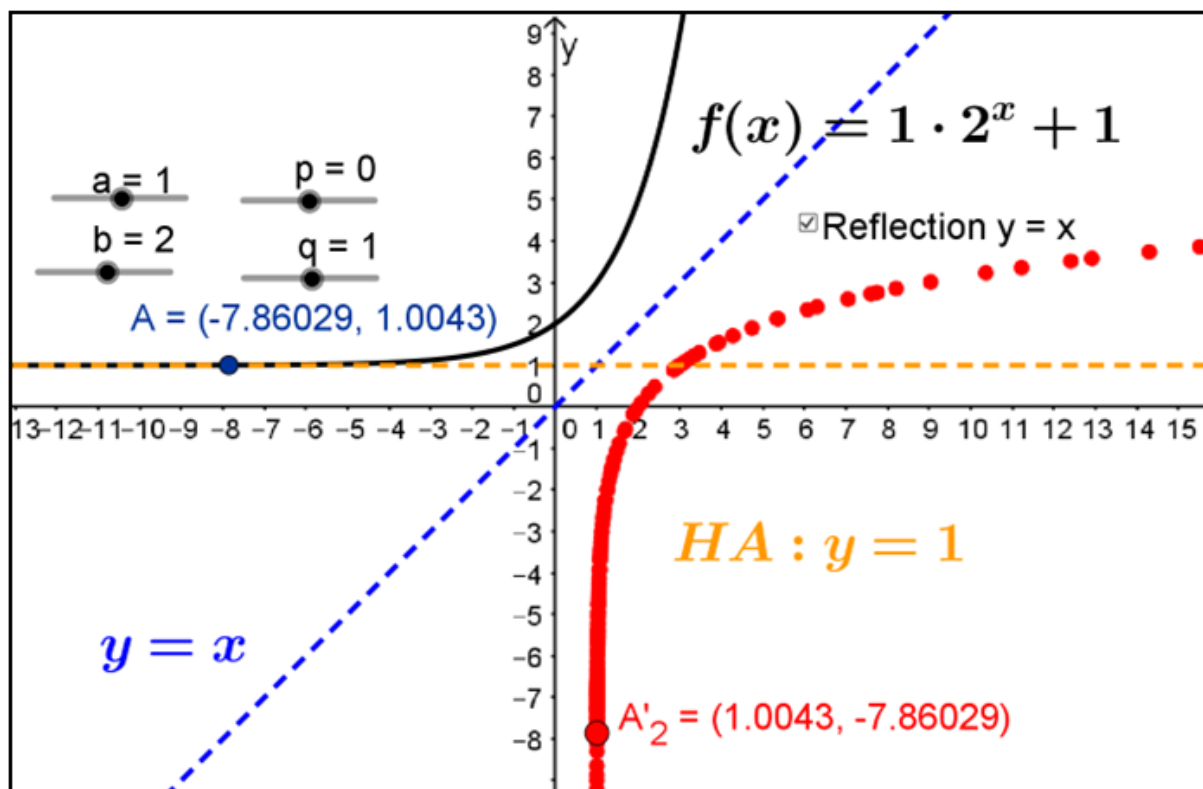


Figure 4: The inverse graph of the exponential function is sketched using the GeoGebra applet

Student FG16 could see the inverse graph of the exponential function as generated by the red dots while observed the mapping of point A to point A'2. Therefore, the dynamic nature of GeoGebra facilitated practical knowledge about the shape of the exponential's inverse graph. Leveraging on the dynamic functionality, students were able to observe the path that the red dots formed. Additionally, the students could observe that point A's y-value remains above 1. This was accomplished by shifting point A to the left, allowing them to observe that point A'2's x-value was greater than 1. Students gained understanding of logic and mathematics through these activities, and they learnt how to

illustrate the inverse graph of an exponential function which aided the understanding of transformations of functions. A student exclaimed that she understood much!

Going further about the notations, student ID8: explained that she could better understand why the sign if $g(x)$ is reflected in x -axis... In her words, she expressed that then I know the negative sign has to stand in front of $g(x)$, and it also means that the new graph is $h(x)$ (potentialities and entablements) Figure 5 demonstrates how Student ID8 used GeoGebra to help her comprehend transformation notation.

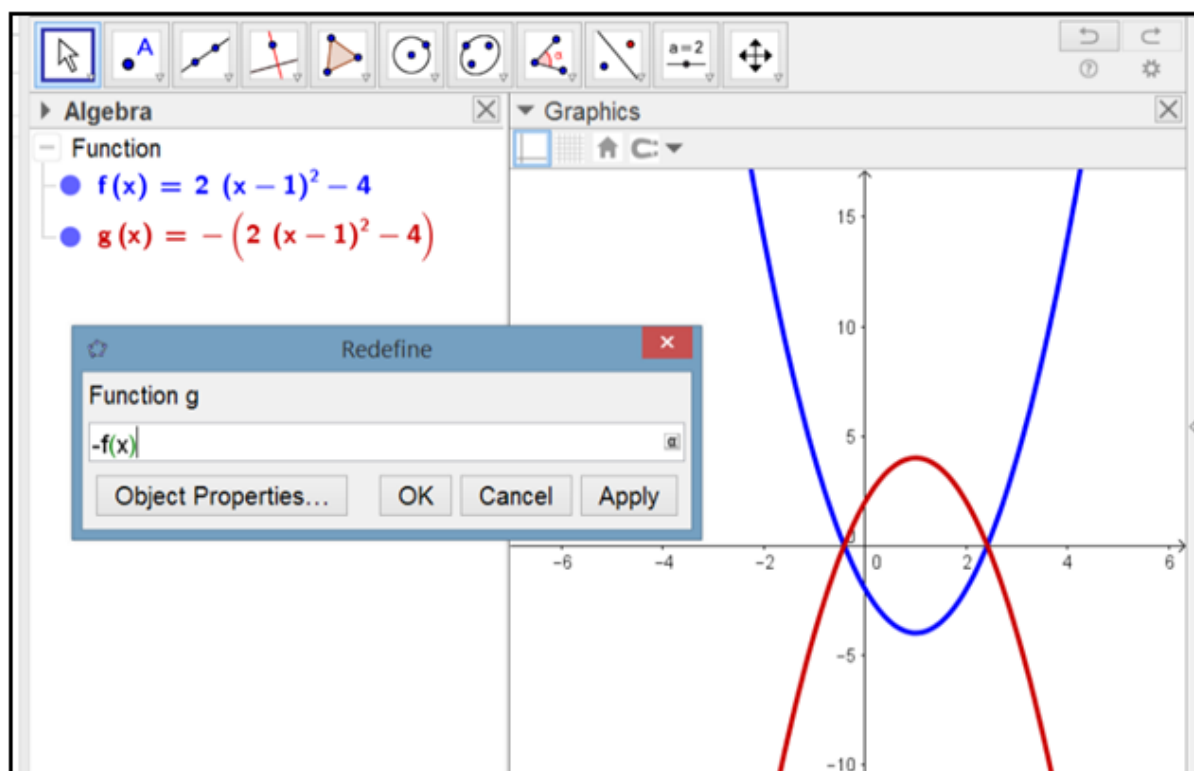


Figure 5: GeoGebra applet to support Student ID8 explanation

Student ID8 described how she was able to connect the algebraic and pictorial representations with the aid of GeoGebra. She also explained how the dialog box helped her to understand that (x) is redefined as g and $g(x) = -f(x)$. The student's response revealed that GeoGebra assisted them in comprehending the impact of the parameters. This supports the suggestion that mathematics should be taught in a way that helps students comprehend how the various parameters affect results (DBE, 2011). Activities involving students using GeoGebra applets are intended to help them visualise algebraic notations graphically and comprehend how words change from the notation format as shown in Figure 5. This activity reflects students' vertical mathematical reasoning. Additionally, by utilising the graphic window's features, the students were able to visually grasp problems and connect them to GeoGebra's algebraic window. According to Sattar et al. (2015), this exercise demonstrates how GeoGebra aided in the establishment of coherence at the deeper structure level of the notation $-(x)$.

During the in-depth (ID), two students explained their in-school learning experiences as follows:

Student ID9: explained that formulas were just given which encouraged rote learning with no meaningful knowledge of the content of instruction.

Student ID6 expressed how the best she could do was to memorise the formulas explaining that if it is reflected in $y = x$ "then you must remember to swap it around".

During the computer session, the students were aware that while dealing with the function's inverse, the x - and y -values switch places. They were unable to provide an explanation, and the majority of them demonstrated in the pre-test that they did not comprehend the significance of the transformation of the notation $f^{-1}(x)$. It's possible that this was taught as an algorithm or process to follow because the students were unable to understand why the x - and y -values interchanged. This resulted to rote learning with little or no grasp and comprehension of the subject matter.

During the focus group interviews (FG), a few students described their educational experiences as being spoon-fed with no concrete learning (Student FG4), lacked meaningful engagement 'you don't know why you are solving it but had to just do it' (Student FG13). Furthermore, students FG4 and FG13 acknowledged that some concepts and topics were taught utilising the usual teaching method, while others admitted that they were simply presented without any comprehension of the notion. As a result, students were only trained to memorise the rules and algorithms, and these ideas were presented as social knowledge. This learning

experiences may be the reason why students completed the pre-test as they were unable to recall that it was social knowledge (Kamii & Ewing, 1996). The responses gathered from the sampled students in the focus groups and in-depth interviews demonstrate how GeoGebra's visual affordances enabled students the chance to learn both logically and aided by visualisation resulting into confidence and deep learning.

Discussion of Findings

The computer lab observations showed that instrumental orchestrations are important when utilising GeoGebra to teach and learn mathematics. Function transformation computer labs favoured guide-and-explain orchestration. As students use computers, the teacher walks around the classroom to provide guidance and explanation (Drijvers et al., 2013a). Furthermore, while students could work autonomously without help, Amineh and Aslin (2015) argued that social constructivism facilitators help students, but as they learn independently, they diminish their support. Students valued the realistic or experienced context, guidance, investigation, and participation, according to the results of the in-depth and focus group interviews as well as computer lab observations. This finding aligns with Ntshangase et al. (2024) which revealed that students appear to perform better academically and have a greater interest in learning mathematics when instructed using GeoGebra software rather than conventional teaching techniques. Effective education takes more than GeoGebra, say quantitative and qualitative statistics. Thus, RME-based instructional design uses GeoGebra. Guided reinvention, realistic interaction, and activity principles using GeoGebra were covered. Student quantitative and qualitative data showed they strongly believed GeoGebra-designed exercises helped.

Students expressed the need for real-world math applications. Students explained that investigating, imagining, and applying arithmetic helped them learn. GeoGebra instruction began with direction-based exploration. Students did GeoGebra activities. The researcher created these activities to teach function transformations. RME's guided reinvention technique lets students discover math (Gravemeijer & Terwel, 2000). The specified activities improved conceptual understanding, according to DBE (2014), Barnes (2008), etc. Conceptual understanding should be taught, not algorithms. In in-depth and focus group interviews, most students said GeoGebra helped them in all areas and notably in function transformations. At the interaction stage in teaching with GeoGebra, the

results showed students liked the exposure, as shown through the in-depth and focus group sessions, which revealed that students believed their interactions fostered confidence and helped them encourage each other. It was also nice that GeoGebra in the computer lab let them debate screen images. This finding shows that GeoGebra allowed students to interact, which is a crucial aspect of social constructivism (Ertmer & Newby, 2013). Sousa (2006) asserts that students can retain 90% of information in 24 hours, provided they can communicate. Çakiroğlu et al. (2024) argue interaction helps students solve complex problems. Questions involving table completion, deductions, and explanations increased activity. These GeoGebra instructional design projects were popular with students. According to Hacıomeroglu et al. (2009), drawings, figures, and diagrams do not assist students in learning. According to their research, GeoGebra worksheets with static illustrations must be dynamic.

Students choose a GeoGebra teaching method in stage three. Seventy percent of the students involved in the in-depth interviews and sixty five percent of students in the focus group interviews expressed that GeoGebra can be utilized to teach arithmetic in all three ways. Ten percent of students involved in the in-depth interviews indicated only the teacher should use it, while twenty-two percent of students interviewed in the focus group enjoyed learning using GeoGebra. Most students interviewed felt that the GeoGebra methods are useful for arithmetic instruction. In-depth and focus group interviews showed students preferred prepared GeoGebra applets for function transformation activities. Students also choose to type transformation properties and functions individually. Student GeoGebra use suggests instrumental genesis (Trouche, 2004). Transformation was taught, and students chose relevant functions from their individual activities. Students drew GeoGebra graphs to examine data and answer research questions in the statistics assignment. Instrumentalisation is seen as student artifact mastery (Ndlovu et al., 2011; Trouche, 2004).

The study findings show that students visualize topics, which has also been proved to be useful for learning mathematics using GeoGebra and allows for concretisation of concepts, mitigates difficulties, facilitates hands-on learning experiences, promote retention of information through repetitive use, and makes mathematics interesting (Zhang et al., 2023). Furthermore, the need for instrumental orchestrations is further strengthened when technology such as GeoGebra is used in mathematics classrooms by teachers (Drijvers et al., 2013; Xu & Gong, 2024). Beginner learners prefer prepared

applets, suggest Hohenwarter et al. (2008). The results confirm De Villiers's premise that students can investigate, learn, comprehend, and speculate without knowing the program. De Villiers (2007) suggested using ready-made applets for sophisticated activities like dragging and clicking animation and construction buttons to analyze altered function input and output values. Students can make mathematical conjectures using quasi-empirical methods (De Villiers, 2004). Students liked one GeoGebra approach for its benefits. Even though it may be impossible, the results suggested GeoGebra should include students' choices. These findings support Hohenwarter et al. (2008) on teacher- and student-centered GeoGebra use. Thus, combining multiple orchestration methods (teaching styles) aided by technology rather than overusing one keeps students engaged and motivated.

Conclusion

Based on the findings of the study, most students valued the use of GeoGebra's applications for teaching and learning transformation and selected relevant functions in mathematics from their personalised activities, such as graphs and sketches, though they preferred using pre-made GeoGebra applets for certain tasks independently while acquiring physical and logico-mathematical knowledge with transformations of functions and graphs. This study strengthens the need for the use of GeoGebra for effective teaching and learning of mathematical knowledge of transformations in functions and graphs within the context of students entering the university.

Recommendation

The following are recommendations informed by the conclusion drawn from this study:

1. Mathematics teachers should engage a guided interaction with GeoGebra during lessons, which helps scaffold their understanding and promotes independent exploration of mathematical concepts, teachers should regularly incorporate ready-to-use GeoGebra applets into instructional activities involving transformations of functions
2. Student-centered GeoGebra projects that require students to create and analyze their own GeoGebra constructions in graphs, sketches, and dynamic visualizations should be encouraged to foster student agency and personalized learning, allowing them to discover mathematical properties, develop critical thinking, and deepen their understanding through inquiry-based learning.
3. Mathematics teachers should constantly design learning activities that explicitly link concrete manipulations such as dragging slides with abstract

reasoning in formulating transformation rules using GeoGebra which is deemed essential for students transitioning from secondary to university-level mathematics.

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